The inversion of measurement.
The pivotal role of space concepts in physics

Rainer Gruber

"The contrary is false as well"

In memoriam Ute Stammberger, my beloved wife, who died too soon

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ABSTRACT

As a contribution to the ongoing debate on the relation between Einstein's general relativity and quantum theory we revitalize Eddington's conjecture that measuring is not a process of comparison with some presumed external entity dubbed Nature but a process of identification of the classical measuring entities with the variables of a space concept that serves to encode the condition of the possibility to measure. By analyzing the mathematics of general relativity, elementary particle physics, electrodynamics and quantum mechanics we show this conjecture to provide the key for understanding the existence of distinct realms in physics, their mutual relation and their success.

Note: The author of this paper passed away before completion of the writeup. Therefore, some known issues in transfers of mathematical formalisms to expressions in physics theory could not be resolved, and some flaws could not be corrected any more. Nevertheless, the general idea, and several methodological approaches of cross-feeding algebra of tensors and spinors with concepts in theoretical physics, may be stimulating to the reader.

1. Introduction

1.1. Tracing the physiognomy of Riemannian space: Eddington's principle of identification

By discarding the restrictions of Euclidean geometry which proved to be too restrictive to reproduce the Perihelion shift of Mercury the new concept of Riemannian space allowed Einstein to explain the *Perihelion shift* and moreover to predict the *redshift* of light and the *deflection* of light in the gravitational field of the sun.

Eddington [1] in his seminal 1923 book *The Mathematical Theory of Relativity* [2] unfolded the idea that the structure of Einstein's General Relativity and its successes are completely determined by the structure of Riemannian space as soon as the metric $g_{\mu\nu}$ required to be able to measure gets *identified* with the 2-rank tensor $G_{\mu\nu}$ derived from the Riemann-Christoffel tensor. This requirement constitutes the *condition of the possibility to measure*. It is identical with Einstein's 1^{st} field equation. On the assumption that forcefree objects in General Relativity move along geodesics the two most impressive results of early General Relativity - the Perihel shift of Mercury and the deflection of star light in the gravitational field of the Sun - are correctly derived from this equation within a few mathematical steps.

The bold step that Einstein took and that rendered possible these successes of early General Relativity was the ingenious idea to postulate the existence of a gravito-inertial field to be identified with the metric $g_{\mu\nu}$ of Riemannian space.

For these early successes irrelevant but nevertheless important to complete the theory Einstein went on to *identify* the contravariant observational entities compiled in the energy-stress tensor $T^{\mu\nu}$ with the covariant variables of the space concept contained in $G_{\mu\nu}$. This marks Einstein's 2^{nd} field equation.

1.1.1. A new interpretation of measurement

The stunning ease of the derivation of the fundamental observational facts of General Relativity from a simple commitment about measuring within the basic space concept led Eddington to fundamentally revise the concept of measurement.

The traditional view of measurement as a process of comparing the outcome of observations with preexisting entities of an extraneous entity dubbed Nature according to the conjecture of Eddington should be replaced by what he calls a *principle of identification*. The entities measured by observers - instead of being subjected to a process of comparison - get identified with the variables describing an underlying space concept, viz. Riemannian space. The concept of measurement in the traditional sense gets replaced by the *principle of identification*.

The radically new interpretation of measurement says: contrary to the traditional claim no reference to an extraneous entity like Nature is involved in any measurement. What physicists see as a result of measuring is reflecting not Nature but is reflecting the *condition of the possibility to measure* that has been encoded in the underlying space concept.

The space concept in this view is serving two purposes: (i) it offers an invariant that plays the role of a measuring stick, $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, and (ii) it allows to encode the *condition of the possibility to measure*. This condition is constituting the equation of motion. A measurement whence while fulfilling the *condition of the possibility to measure* will confirm the equation of motion. According to the conjecture of Eddington this intrinsic relation provides the key to explain the splendid successes of early General Relativity.

1.1.2. A series of strange identifications pervades the edifice of physics

A century later a closer look reveals a whole series of such strange identifications pervading the edifice of theoretical physics, identifications physicists have become familiar with. Nothing in these identifications seems to be rationally placeable:

- In *elementary particle physics* the fundamental leptons and quarks get identified with the components of spinors. Spinors are the mathematical parameters that allow to span complex flat space by base vectors of length zero.

- In *Maxwell*'s theory of electro-magnetism the electromagnetic field tensor $F_{\mu\nu}$ gets identified with the bivector of real flat space with the *electric* and *magnetic* field representing its polar and axial components.

[1] estimated to be *the most distinguished astrophysicist of his time* (Chandrasekhar 1983,39)

[2] First published in 1923, the 2^{nd} edition of 1924 experienced its eleventh reprint in 1975.

- In Quantum Mechanics (QM) the *quantum mechanical* objects whose existence gets confirmed by our experimental data get identified with *wave functions* representing the mathematical support of the generators of translations combined with Galilei transformations.

Though rationally inaccessible nobody is wondering anymore about these identifications.

1.2. The aim of this paper

The aim of this paper is to show that the conjecture of Eddington derived from the analysis of General Relativity as well applies to the other realms of physics: elementary particle physics, Maxwell's electrodynamics, quantum mechanics.

Indeed the different realms of physics historically have evolved as qualitatively distinct because they are built each one on another space concept. The consistency of the respective space concept guarantees the consistency of the theoretical description. Consistency with the experimental findings is established because the objects that get found experimentally are entities emerging from the space concept.

In each of the above mentioned realms the experimental and theoretical efforts end up in the astonishing insight that the resulting theoretical description is *tracing the physiognomy of the underlaying space concept* when this concept has become equipped with the appropriate *condition of the possibility to measure*.

We will begin with shortly sketching the considerations of Eddington related to General Relativity (sect.2). We then concentrate on *elementary particle physics* where the insight of Eddington gets its most obvious realisation (sect. 3.1). After a short excursion to Maxwell's electrodynamics (sect. 4) we show how Quantum Mechanics fits into this scheme (sect.5).

2. Einstein's general relativity

The following presentation is taken from the book of Sir Arthur Stanley Eddington *The Mathematical Theory of Relativity*, 1975, first ed. 1923. We try to avoid mathematical subtleties, that tend to obscure the perspecuity of the derivation. Interested readers may consult the original book.

2.1. Identification as a key to understand General Relativity

General Relativity (GRT) is based on the concept of Riemannian space. This space concept offers an invariant length element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{1}$$

with $g_{\mu\nu}$ the metric tensor. The existence of a *metric tensor* in the framework of General Relativity is believed to be indispensable to perform a measurement [3]. The only tensor available in Riemannian space that could serve this purpose is derived from the fundamental Riemann-Christoffel tensor $B^{\rho}_{\mu\nu\epsilon}$ by the mathematical operation of contraction:

$$G_{\mu\nu} = B^{\rho}_{\mu\nu\rho} \tag{2}$$

$G_{\mu\nu}$ is called the Einstein tensor. The *condition of the possibility to measure* is established by *identifying* the metric tensor $g_{\mu\nu}$ with the Einstein tensor

$$G_{\mu\nu} = \lambda g_{\mu\nu} \tag{3}$$

with λ a proportionality constant. [4] This is the 1^{st} Einstein field equation.

The fact that for all practical applications λ is very small allows to write eq.(3) as

$$G_{\mu\nu} = 0 \tag{4}$$

This equation is referred to as Einstein's *law of gravitation*.

[3] Note the difference to *elementary particle* physics and *quantum mechanics* where no such tensor is available

[4] "Tensors not containing derivatives beyond the second must necessarily be compounded from $g_{\mu\nu}$ and $B^{\epsilon}_{\mu\nu\sigma}$ so that, unless we are prepared to go beyond the second order, the choice of a law of gravitation is very limited, and we can scarcely avoid relying on the tensor $G_{\mu\nu}$. Without introducing higher derivatives, which would seem out of place in this problem, we can suggest as an alternative to $G_{\mu\nu} = 0$ the law $G_{\mu\nu} = \lambda g_{\mu\nu}$ " (Eddington 1923,81)

Inserting the condition of the possibility to measure, eq.(4), into the invariant length element eq.(1) within a few steps [5] leads to the solution known as the *Schwarzschild* metric (Edd 1923,85) with m an integration constant.

$$ds^2 = -\gamma^{-1} dr^2 - r^2 d\theta^2 - r^2 sin^2\theta \, d\Phi^2 + \gamma \, dt^2 \qquad \text{with} \qquad (\gamma = 1 - \frac{2m}{r}) \tag{5}$$

2.2. The early successes

Eq.(5) immediately implies a displacement of the Fraunhofer lines in a gravitational field, the so-called gravitational redshift of light (Edd 1923,91). Assuming the atoms in some region to be at rest, $dr, d\theta, d\Phi = 0$ gives

$$ds^2 = \gamma \, dt^2 \qquad \text{with} \qquad (\gamma = 1 - \frac{2m}{r}) \tag{6}$$

Accordingsly the times of the vibrations of the differently placed atoms will be inversely proportional to $\sqrt{\gamma}$. The observations indeed indicate this redshift to be found in the data.

Assuming objects in curved space to move on *geodesics* [6], i.e. inserting eq.(5) into the equation for a geodesics

$$\frac{d^2 x^\mu}{ds^2} - \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0 \tag{7}$$

immediately reproduces the miraculous effects thad made up the fulminant successes of early General Relativity that made Einstein become famous: the Perihelion shift of Mercury and the deflection of light in the gravitational field of the sun.

The 2^{nd} Einstein field equation

$$G^{\mu\nu} - 1/2 g^{\mu\nu} G = 8\pi\kappa T^{\mu\nu} \tag{8}$$

determines how to identify the contravariant measuring entities of classical mechanics - as compiled in the energy-stress tensor $T^{\mu\nu}$ - one by one with the covariant curvatures assembled in the Einstein tensor $G_{\mu\nu}$. A constant κ $[cm/g]$ appears to readjust the dimensions of the covariant space variables $[cm]$ to the contravariant dimensions used by experimentalists $[g]$. It turns out that κ is Newton's gravitational constant.The lhs is divergence free by mathematical identity. By this identification the energy-stress tensor automatically gets divergence free.

It may be noted that the early successes are derived without making use of the 2^{nd} Einstein field equation. This equation gets needed to establish the connection to continuous matter and to address Newton's theory (Edd 1923,p.101).

Both Einstein equations reflect the fact that the aim of physicists is to measure. What these equations do is to encode the condition of the possibility to measure in Riemannian space, eq.(3), and to identify the measuring entities with the variables of the space concept, viz. the curvatures compiled in the Einstein tensor, eq.(8).

The bold step Einstein took was to *identify* the gravitational field addressed by Newton's theory with the metric $g_{\mu\nu}$ of Riemannian space. This identification is not expressed by a mathematical equation. It constitutes the fondament of General Relativity.

2.3. A new conceptualization of measurement

The ease with which the fundamental and gloriously confirmed new results of General Relativity are derived from simple commitments about measurement led Eddington to introduce his *Principle of identification*. This principle presents a key to understand this phenomenon. It suggests a radically new interpretation of measurement.

Traditionally a measurement is thought to allow to compare the outcome of a theory with the features of some metaphysical extraneous entity dubbed Nature. This is conjectured to be a process of ever better approximation to reveal the nature of Nature.

In contrast measurement according to Eddington means the *identification* of the entities used by observers with the variables of an adequately chosen space concept. At the end the requirement of *consistency* acts as the invisible hand that guides the theory and the experiment to trace the physiognomy of the underlying space.

[5] with the following boundary conditions specified: (i) *spatial spherical* symmetry to address an isolated object; (ii) postulating a *static* solution as an essential condition to derive any analytical solution at all; (iii) invariance against *time reflection*; (iv) the invariant length ds^2 gets required to approach the Minkowski metric at $r \to \infty$.

[6] This would be not adequate neither for elementary particle physics nor for Quantum Mechanics

2.4. The objects emerge from the condition of the possibility to measure

How is consistency of the theoretical implications with the observational results being effected? The solution of this mystery lies in the fact that the space concept provides the mathematical structures to be identified with "objects". These objects by their very nature emerge from the *condition of the possibility to measure* encoded in the space concept - together with the mutual interactions these objects undergo. These objects astronomers trace with their measuring appliances.

The consistency of the space concept induces the consistency of the theoretical implications with the observations made on objects that appear to be predefined in the metric.

2.4.1. The emergence of Keplerian orbits

The set of differential equations given by inserting the Schwarzschild metric eq.(5) into the geodesics eq.(7) allows to isolate an invariant of the equation of motion

$$\gamma \frac{dt}{ds} = c \qquad \text{with c an integration constant}\,, \qquad (\gamma = 1 - \frac{2m}{r}) \tag{9}$$

indicating that the coordinate time dt near the Schwarzschild radius $r_S = 2m$ will begin to explode in comparison with the invariant length element:

$$dt = c\frac{ds}{1 - \frac{2m}{r}} \tag{10}$$

Eliminating dt and ds and switching to a coordinate $u = 1/r$ leads to the equation of the Keplerian orbit which classically (i.e. far from the Schwarzschild radius) would have been interpreted as the motion of a planet in the gravitational field of a central object of mass m:

$$\frac{d^2u}{d\Phi^2} + u = \frac{m}{h^2} + 3\,mu^2 \tag{11}$$

with $h = r^2 \frac{d\Phi}{ds}$ another integral of the motion. This resembles the classical equation of a planetarian orbit with the coordinate differential dt in the constant h replaced by the invariant length element ds. The additional factor $(1 - 2m/r)$ in the metric appears as an additional term $3\,mu^2$ in the equation of the orbit eq.(11) determining the *Perihelion shift* [7] of planetarian orbits [8].

2.4.2. The identification of Mercury as a disturbance in the metric

Both objects, the *planet* as well as the Keplerian *central object*, are an imagined extrapolation within the Newtonian picture, the central object being represented by the heavy mass m. These objects emerge from the space concept as soon as it becomes equipped with the *condition of the possibility to measure* eq.(4). The gravito-inertial field, being identified with the metric $g_{\mu\nu}$, constitutes the interaction these emerging objects undergo.

The inability to reproduce the perihelion shift of Mercury within the frame of Newtonian physics showed that the restrictions imposed by Euclidean geometry to admit rigid geometrical structures were too strong to be able to cover the observational reality measuring appliances would show. Skipping these restrictions immediately unleashed the modification $(1 - 2m/r)$ within the metric responsible for the Perihelion shift of the orbits identified within the metric.

Tracing the calculation of the Perihel shift of Mercury shows that it is not the planet Mercury that measuring appliances are measuring. What we measure is a disturbance in the metric. The influence of this small disturbance of the metric shows that the astronomers with their observational appliances trace the metric of the underlying space concept with all its ramifications. Euclidean space with its restrictions to rigid objects is a too rigid a space concept to be able to reproduce the results of observations. Riemannian space is not a more complicated space but a space concept freed of these restrictions.

[7] which in the case of Mercury by century-long accumulation has become remarkable for astronomers (Edd 1923,88).

[8] The classical derivation is using dt instead of ds. dt is a complete differential necessary to represent a coordinate but not invariant. ds is invariant but not a complete differential. This difference is insignificant at this point of discussion.

2.4.3. *The emergence of BH's within the metric*

Long after Eddington's reflections on Einstein's General Relativity it was recognized that the most prominent disturbance given by the factor $(1-2m/r)$ indicating a spurious singularity of the metric could be adressed as Black Holes (BH), objects [9]. now believed to reside inside nearly every major galaxy. The features showed by and defining BH's are taken from the ramifications the Schwarzschild metric eq.(5) offers.

2.5. General Relativity is a statement about measurement

General Relativity doesn't describe an outer world dubbed Nature. General Relativity is a statement about measurement. It is a statement on how to encode the *condition of the possibility to measure* in the space concept.

The space concept when married with the *condition of the possibility to measure* makes emerge the objects of the theory. Einstein's bold step to identify a postulated physical gravitational field with the metric $g_{\mu\nu}$ [10]. provides the bridge for the successfull identification of the objects observed in astronomy with disturbances of the metric. This step defines the interaction of the astronomical objects.

Both, observations and their theoretical framing, appear to be tracing the physiognomy of the space concept. The birth of black holes in the metric and their identification in astronomical observations is a hint to the relevance of Eddington's conjecture.

The internal consistency of the mathematical space concept finally not only does guarantee the internal consistency of the physical theory but also its consistency with the results of observations, since the objects of the theory emerge from the space concept. The key to and solution of this enigma is the encoding of the *condition of the possibility to measure* within the space concept.

3. Elementary particle physics

At the heart of our paper is the discovery that elementary particle physics with all its ramifications as compiled in the Standard Model (SM) is tracing the physiognomy of flat space when this space is extended to complex. The structure of a flat complex space has been investigated by *Cartan* (1938) in his standard book *The theory of spinors*. [11] We give a short summary of the mathematical base followed by the consequences of the identifications that make it become the base of elementary particle theory. [12]

3.1. Flat space taken to be complex

3.1.1. *The defining quadratic form of a complex space*

Real n-dimensional flat spaces E_n are constituted by the existence of a quadratic form [13] $\Phi = (x_r^1)^2 + \ldots + (x_r^n)^2$. This form is invariant under rotations and translations and thus capable to serve as the representation of a *measuring rod*.

Cartan has chosen the quadratic form F to characterize a *complex* space:

$$F \equiv z_1 z_{1'} + \ldots z_i z_{i'} + \ldots + z_\nu z_{\nu'} + z_0^2 \tag{12}$$

A prominent feature is a pairing of the variables: the indices i, i' represent single complex dimensions grouped into pairs. The variable z_0 is designating an *unpaired* dimension taken to be real by assumption. The transition to real space of dimension $n = 2\nu + 1$ is done by choosing $z_{i'}$ to be the complex conjugate of z_i. Setting the unpaired coordinate z_0 identically to zero then leads to a real space of dimension $n = 2\nu$.

[9] The transition from black holes to the Keplerian central objects needs a) a transition to isotropic coordinates that guarantee that a measurement with a rigid stick (ds=constant) effected in different directions gives the same result (Edd 1923,93) and b) the transition to continuous matter (Edd 1923,101)

[10] with g_{00} appearing to represent the Newtonian potential

[11] Throughout this paper we will refer to this book by abbreviating (C 100) meaning (Cartan 1938, 100)

[12] We try to avoid mathematical subtleties, that tend to obscure the perspecuity of the derivation. Interested readers may consult the original book.

[13] The subscript *r* denotes coordinates in real space.

3.1.2. *Compound indices*

The transition to complex automatically makes the orthogonal unit base vectors to become isotropic, i.e. vectors of length zero. The appearance of *isotropic* vectors according to Cartan is the paradigmatic geometrical foundation of the existence of spinors.

The coefficients needed to span the space by isotropic vectors constitute the components of a spinor ξ_α. Their explicit geometrical construction by a recursive procedure shows that the index α becomes a *compound* index, composed by single indices i, i' referring to the basic complex dimensions of the vector space.

A successive nesting of linear forms η_α:

$$\eta_0 \equiv \xi_0 x^0 + \sum_k \xi_k x^k = 0 \qquad \text{(k=1} \ldots \nu) \tag{13}$$

$$\eta_i \equiv \xi_0 x^{i'} - \xi_i x^0 + \sum_k \xi_{ik} x^k = 0 \tag{14}$$

$$\eta_{ij} \equiv \xi_i x^{j'} - \xi_j x^{i'} + \sum_k \xi_{ijk} x^k = 0 \tag{15}$$

$$\eta_{ijk} \equiv \xi_{ij} x^{k'} + \xi_{jk} x^{i'} + \xi_{ki} x^{j'} - \xi_{ijk} x^0 + \sum_h \xi_{ijkh} x^h = 0 \qquad \text{etc.,} \tag{16}$$

leads to coefficients ξ_α with the single indices i, i' agglomerated to ever higher nested *compound* indices $\alpha = i_{k_1} i_{k_2} \ldots$ reflecting the nesting status:

$$\xi_0 \xi_{ijk} = \xi_i \xi_{jk} - \xi_j \xi_{ik} + \xi_k \xi_{ij} \tag{17}$$

$$\xi_0 \xi_{ijkh} = \xi_{ij} \xi_{kh} + \xi_{jk} \xi_{ih} + \xi_{ki} \xi_{jh} \qquad \text{etc.} \tag{18}$$

The procedure leads to 2^ν coefficients ξ_α that are addressed to be the components of the spinor ξ. In contrast to the components of vectors the number of spinor components grows exponentially with the dimension as $n = 2^\nu$.

The index α of the spinor ξ_α hereafter is a *compound* of single indices that are related to the coordinate axes. A spinor for $\nu = 2$ is composed by 4 components (ξ_0, ξ_1, ξ_2, ξ_{12}); a spinor for complex dimension $\nu = 3$ has 8 components (ξ_0, ξ_1, ξ_2, ξ_3, ξ_{12}, ξ_{13}, ξ_{23}, ξ_{123}) etc. Besides the single component ξ_0 and the single indexed components ξ_i $(i = 1, 2, \ldots \nu)$, which formally ressemble the component structure familiar from vectors, spinors have additional components $\xi_{i_1 i_2 \ldots i_p}$ $(p = 2, \ldots, \nu)$. These have the property of either changing sign or being unaltered under odd or even *permutations* of the indices.

The spinor components are the same whether there is an unpaired dimension or not.

3.1.3. *The defining equation of spinors*

The recursive linear equation system eq.(13)- (18) defining the spinor components ξ_α in a space with isotropic vectors $x_0, x_i, x_{i'}$ $(i = 0, 1, \ldots \nu)$ may be inverted to explicitly show up the spinor components as variables:

$$X\xi = 0 \tag{19}$$

with X a $2^\nu \times 2^\nu$ matrix and ξ a spinor with 2^ν components with a definite sequence of the spinor components α [14] adopted. X is called the *associated matrix* of the vector x.

Let the associated matrices of the *isotropic* basis vectors $\vec{e}^{\,0}$, $\vec{e}^{\,i}$, $\vec{e}^{\,i'}$ be the matrices $H_0, H_i, H_{i'}$. Geometrically they represent a reflection on the hyperplane that is perpendicular to the respective basis vector and contains the origin. Each vector $\boldsymbol{x}$ hence may be represented by the associated matrix X decomposed in terms of reflection operators:

$$X = x^0 H_0 + x^1 H_1 + \ldots + x^\nu H_\nu + x^{1'} H_{1'} + \ldots + x^{\nu'} H_{\nu'} \tag{20}$$

[14] e.g. for $\nu = 3$: 0,1,2,3,12,13,23,123

3.1.4. Totally antisymmetric p-vectors

Vectors are members of a set of *totally antisymmetric objects* that inhabit flat space. These objects called p-vectors consist of the totally antisymmetric sum of products of the components of p vectors (C 15,83). Following Cartan we denominate the associated matrix of a p-vector by $\underset{(p)}{X}$. It is defined to be one for $p = 0$ and to represent the associated matrix X for $p = 1$.

The decomposition (20) illuminates the central role that *reflections* are playing in a representation based on spinors.

3.2. Towards a unified picture of interactions

3.2.1. The fundamental polar and its irreducible components

What makes this complex space attractive for physicists and indeed makes it become the foundation of a new realm of physics is the appearance of a new entity, the *fundamental polar*

$$\xi^T C \xi \tag{21}$$

It consists of the product of two spinors mediated by a characteristic matrix $C = (H_1 - H_{1'}) \dots (H_\nu - H_{\nu'})$ built by the reflection operators $H_i, H_{i'}$. The fundamental polar remains unchanged under a rotation, but under a reflection it is reproduced multiplied by $(-1)^\nu$. If ν is even this form is invariant under rotations and reversals. If ν is odd it changes sign under a reversal.

Under rotations or reflections the fundamental polar is not irreducible. To decompose it into its irreducible components we consider the tensor $\xi'_\alpha \xi_\beta$. By decomposing this tensor into irreducible representations a series of tensors $\mathcal{T}_p$ emerges

$$\mathcal{T}_p = \xi^{T'} C \underset{(p)}{X} \xi \tag{22}$$

with $\underset{(p)}{X}$ denoting the associated matrix of an antisymmetric multivector of rank p.

These tensors are irreducible against rotations and reflections. Together they constitute the decomposition of the fundamental polar into its irreducible components. For $p = 0$ we rediscover the fundamental polar.

Containing *reflection operators* the matrix C as well as the associated matrix $\underset{(p)}{X}$ make up the dynamical content of $\mathcal{T}_p$ physicists are interested in.

3.2.2. Reflection operators acting as creation and annihilation operators

The reflection matrices obey the rules:

$$H_0^2 = 1,\ H_0 H_k = -H_k H_0,\ H_0 H_{k'} = -H_{k'} H_0 \qquad (k \neq 0) \tag{23}$$

$$H_i H_k = -H_k H_i,\ H_{i'} H_{k'} = -H_{k'} H_{i'} \tag{24}$$

where the last equation for $i = k$ means $H_i^2 = H_{i'}^2 = 0$ [15] . For the *conjugate pairs* we get:

$$H_i H_{k'} + H_{k'} H_i = \delta_{ik} \tag{25}$$

We may write the rules eq.(23) $-$ (25) in the form of commutators:

$$[H_i, H_j]_+ = [H_{i'}, H_{j'}]_+ = 0 \tag{26}$$

$$[H_i, H_{j'}]_+ = \delta_{ij} \tag{27}$$

showing that the H_i' and H_i are *creation* and *annihilation* operators .

[15] It is helpful to know that $H_0 = H_0^T$ and $H_{i'} = H_i^T$.

3.2.3. The action of reflection operators on spinor components

It is instructive to become familiar with the way reflection matrices unfold their annihilation and creation power on the spinor components ξ_α (Cartan 1981,84).

- The operator H_i $(i = 1, 2, \nu)$ replaces by zero those components of ξ_α for which the compound index α includes the simple index i, and adds this index to the ξ_α which do not already contain it; e.g. H_3 transforms ξ_{45} into ξ_{453} and ξ_{23} becomes zero.
- The operator $H_{i'}$ makes zero those components ξ_α for which α does not contain an i and suppresses the index i in those for which α does contain the index i which must first be brought to the last position in the compound index α; for example $H_{3'}$ makes ξ_{45} zero and transforms $\xi_{134} = -\xi_{143}$ into $-\xi_{14}$.
- We easily deduce that $H_i H_{i'} \xi_\alpha$ results in ξ_α or zero depending on whether α does or does not contain i, and vice versa: $H_{i'} H_i \xi_\alpha$ is zero or ξ_α depending on whether α does or does not contain i. Thus $(H_{i'} H_i - H_i H_{i'})\xi_\alpha$ results in $-$ or $+$ ξ_α depending on whether α does or does not contain the single index i.

The reflection operators $H_i, H_{i'}$ thus transform one component of a spinor into another component.

3.2.4. Fundamental fermions identified with the components of one spinor

This is the mathematical base for the physical intuition to identify fundamental fermions with the components of one spinor. The reflection operators then combine the destruction of one particle with the creation of another one. The Cartan tensors

$$\mathcal{T}_p = \xi^{T'} C \underset{(p)}{X} \xi \tag{28}$$

by this identification get the status of *interaction Hamiltonians*. The interaction is effected by reflection operators acting as creation and annihilation operators on spinor components. These tensors control the ratio in which particles represented by spinor components are transmuted into one another.

3.3. Physical harvest

3.3.1. The emergence of the Clifford algebra and Dirac's equation

Switching from an isotropic to the *orthonormal* system of *real* unit vectors $(\vec{e}_r^{\,i} \vec{e}_r^{\,k}) = \delta_{ik}$ we get the real equivalents A_r of $H_i, H_{i'}$. As an example we show the definition for $\nu = 2$:

$$A_1 = H_1 + H_{1'} \tag{29}$$

$$A_2 = i(H_1 - H_{1'}) \tag{30}$$

$$A_3 = H_2 + H_{2'} \tag{31}$$

$$A_4 = i(H_2 - H_{2'}) \tag{32}$$

Using rules (23) till (27) we obtain the commutation rules

$$A_i A_k = -A_k A_i \qquad (i \neq k); \qquad (A_i)^2 = 1 \tag{33}$$

These operators hence form a *Clifford algebra* (Cartan 1981,83).
The decomposition (20) in terms of real basic vectors then reads

$$X = x^0 H_0 + x^i A_i \qquad (i = 1, \ldots, 2\nu) \tag{34}$$

and the defining equation for spinors eq.(19) in a space with no unpaired coordinate becomes

$$A_i x^i \xi = 0 \tag{35}$$

Taking space to be the momentum space the defining equation for spinors eq.(35) reads

$$A_i p^i \xi = 0 \tag{36}$$

For ($\nu = 2$ viz. $n = 4$) this is the *Dirac equation* for a massless fermion

$$\not p \xi = 0 \tag{37}$$

with $\not p = \gamma_\mu p^\mu$ since the A_i like Dirac's $\gamma-$matrices are defined to be the elements of a Clifford algebra.

The reflection operators A_i are the well known Dirac γ-matrices. The associated matrix P of a vector p^μ corresponds to the Dirac nomenclature $\not p = \gamma_\mu p^\mu$. [16]

Eq.(37) shows that the *defining equation* of spinors constitutes the familiar *Dirac equation* [17] in momentum space for massless particles $\gamma_\mu p^\mu \psi = 0$.

Far from being a postulate as it appeared when Dirac came up with his guess of the Clifford algebra to achieve relativistic invariance of the electron's equation of motion the Dirac equation for massless particles obviously is a basic feature of Cartan's geometric spinor theory [18].

We note that these particles necessarily are massless, since with $P\xi = 0$ we get $0 = PP\xi = p^2\xi = m_0^2\xi$. Before the advent of the Higgs field also the fermions of the Standard Model were strictly massless.

3.3.2. Left- and righthanded classes of spinors

The reflection operator H_0 operating along the unpaired dimension z_0 that represents the extension from $E_{2\nu}$ to $E_{2\nu+1}$ acquires a special role: Operating on a spinor ξ_α it gives

$$H_0 \xi_\alpha = \pm \xi_\alpha \tag{38}$$

depending on whether the *compound* index α contains an *even* or an *odd* number of *single* indices.

Let us concentrate on $E_{2\nu}$: we may order the spinor components according to first noting all components with even number of subindices, followed by all components with uneven number of subindices. Then because all the reflection operators $H_i, H_{i'}$ commute with H_0, we can distinguish two groups of semi-spinors, which by rotation get transformed onto itself, representing left-handed and right-handed spinors. For $\nu = 2$ (E_4) we get the semi-spinors of the first type (ξ_0, ξ_{12}) and of the second type (ξ_1, ξ_2). For $\nu = 3$ (E_6) we get the first type to be ($\xi_0, \xi_{12}, \xi_{13}, \xi_{23}$) and the 2nd type to be ($\xi_1, \xi_2, \xi_3, \xi_{123}$).

The spinors in spaces with dimension $n = 2\nu$ hence decompose into two classes of *semi-spinors* whose components have either an *even* or an *odd* number of single indices in their compound index. This decomposition is the mathematical basis for the *existence of two classes* of either left-handed or right-handed fermions.

Any reflection H_i, $H_{i'}$ raises or lowers the number of single indices by one and thus transforms *left-handed* spinors into *right-handed* spinors and vice versa. Any *rotation* then being composed of two reflections projects the members of one class onto itself. This provides for the stability to consider it a class.

In the Standard Model it is considered a dominant feature that the experimentally found fundamental fermions are split according to left- or right-handedness.

3.3.3. The emergence of different types of interaction

Automatically we are led to a series of interactions differing according to the dimension ν of the space the fundamental particles are inhabiting. Being a combination of two spinors and a multivector $\underset{(p)}{X}$ these invariants display the characteristic

[16] For an explicit representation in configuration space see (C 134)).

[17] The introduction of a pseudo-euclidean metric does not change any of the conclusions made here. Cartan in detail unfolded the modifications to be taken into account for the spaces $E_{2\nu+1}$ (C101) and $E_{2\nu}$ (C123). He reproduced the Dirac equation (C134) as an example for the associated matrix of the covariant vector $\partial/\partial x$. Using real basis vectors the reflection operators for $\nu = 2$ are shown to be identical wih the Dirac γ-matrices . Since there is nothing new with respect to our topic we leave it to the interested reader to get all the information about necessary changes from Cartan's book.

[18] In Diracs theory the concentration on the specific case $n = 4$ hides the fact that the spinor components in contrast to vectors obey an exponential dependence 2^ν on the space dimension

form of Yukawa-type interactions familiar from the interaction Hamiltonians in the Standard Model. They obviously offer the blueprint for the interaction Hamiltonians in the Standard Model. We conjecture that the interaction Hamiltonians of QFT owe their exceptional status to the fact that they represent the fundamental Cartan tensors provided by complex flat space within a specific dimension ν.

3.3.4. *Parity violation*

Switching to a base of unit vectors, A will indicate a reflection on the hyperplane π normal to the unit vector a. This operation necessarily is two-valued since we could take either a or $-a$ as the unit vector normal to π.

The effect of a reflection A on the associated matrix X and the spinor ξ is given by the formulae (Cartan 1938,85)

$$\xi' = A\,\xi \qquad X' = -A\,X\,A \qquad \underset{(p)}{X}' = (-1)^p A \underset{(p)}{X} A \tag{39}$$

Since any rotation by an angle θ may be represented by two reflections on hyperplanes that are tilted against one another by an angle $\theta/2$, the effect of a rotation taken as the product of an even number of reflections, $S = A_{2k}A_{2k-1}\dots A_2A_1$ is given by the formula

$$\underset{(p)}{X}' = S \underset{(p)}{X} S^{-1}; \qquad \xi' = S\xi \tag{40}$$

and for a reversal T we get

$$\underset{(p)}{X}' = (-1)^p T \underset{(p)}{X} T^{-1}; \qquad \xi' = T\xi \tag{41}$$

where T is the product of an uneven number $\leq 2\nu + 1$ of matrices associated with unit vectors.

Taking into account these transformation properties and using the property of the matrix C (C89)

$$C \underset{(p)}{X} = (-1)^{\nu\, p + [p(p-1)/2]} \underset{(p)}{X}^T C \tag{42}$$

we get the behaviour of the Cartan invariants $\mathcal{T}_p$ under reflections:

$$[\mathcal{T}_p]' = (-1)^{\nu - p}\,\mathcal{T}_p \tag{43}$$

$\mathcal{T}_p$ whence is a scalar if $(\nu - p)$ is even. It is a pseudoscalar if $(\nu - p)$ is odd (C91).

Some of the interactions $\mathcal{T}_p$ hence show the phenomenon of *parity violation*. One example is $\nu = 2$ and p=1. Experimentally parity violation has been found for the case of weak interactions, with (e,ν_e) and (u,d) the participating particles and the intermediate vector field $\underset{(p)}{X}$ for (p=1) the vector boson field W_μ.

Parity violation whence is an intrinsic property of the spinor representation in complex flat space.

3.3.5. *QED*

Choosing ξ' in $\mathcal{T}_p$ to be the complex conjugate spinor $\xi' = iC\bar{\xi}$ we get (omitting the i) the expression $(C\bar{\xi})^T C \underset{(p)}{X}\xi = \bar{\xi}^T \underset{(p)}{X}\xi$ since $C^TC = 1$. The assignment

$$H_{int} = \bar{\xi}^T \underset{(p)}{X}\xi \tag{44}$$

with $\underset{(p)}{X}$ an associated matrix for $p = 1$ shows a striking similarity with the familiar expression $\tilde{\psi}\gamma^\mu\psi A_\mu$ used for the interaction term in QED, with $\gamma^\mu A_\mu$ the associated matrix of the e.m. potential A_μ.

3.3.6. *The overall set of experimentally detected fermions*

For $\nu = 5$ the 16 leptons and quarks which in the frame of the Standard Model get described as the 16^+ spinor of $SO(10)$ (Wilczek 2006,242) may be neatly identified with the 16 components of the semispinor. They correspond to the fundamental fermions ν, e, u_r, u_g, u_b, d_r, d_g, d_b supplemented by their charge-conjugated counterpart, the antiparticles.

In his 2016 book *Group theory in a nutshell for physicists* Zee presents a realisation of the fundamental 16 fermions that is compatible with the Standard Model, ordered by decreasing charge: $e^+, u, d^c, \nu, \nu^c, d, u^c, e^-$ with the quarks u and d split into the respective three colors.

We may state that for $\nu = 5$ the spinor components do provide for all the fundamental particles needed in the interactions of the Standard Model.

$\nu = 5$ corresponds to the real dimension $n = 10$. We may remind that $n = 10$ is the minimal number of flat dimensions required to mimic the degrees of freedom inherent in the 10 components of the metric $g_{\mu\nu}$ of the 4-dim curved space invoked in General Relativity (Edd 1923,149).

3.3.7. *The expected occurrence of fundamental particles*

Decomposed along handedness the fundamental fermions, leptons and quarks, get identified with the components of semi-spinors. There are $2^{\nu-1}$ semispinor components in a space of dimension ν:

$$\nu = 1 \qquad \text{2 objects:} \quad \xi_\alpha = (\xi_0, \xi_1) \tag{45}$$

$$\nu = 2 \qquad \text{4 objects:} \quad \xi_\alpha = (\xi_0, \xi_1, \xi_2, \xi_{12}) \tag{46}$$

$$\nu = 3 \qquad \text{8 objects:} \quad \xi_\alpha = (\xi_0, \xi_1, \xi_2, \xi_3, \xi_{12}, \xi_{13}, \xi_{23}, \xi_{123}) \tag{47}$$

and so on.

For the moment we will not make any assignment of individual fermions to spinor components since the current experimental knowledge is strongly coined by an analysis in terms of rotations $SU(2)$ and $SU(3)$. We have available the sets (e, ν_e) and (u, d) participating in weak interactions and the sets $(u_r, u_g, u_b, d_r, d_g, d_b)$ participating in strong interactions each complemented by their charge conjugate. The characterization as *weak* and *strong* is referring to an analysis in terms of the generators of $SU(2)$ and $SU(3)$. We did not undertake the translation to a representation in terms of reflection operators nor did we handle the topos of charge conjugation.

3.3.8. $\nu = 4$: *The phenomenon of triality*

For $\nu = 4$ the isotropic vector and the two semi-spinors have an equal number of components (eight). This leads to the phenomenon of *triality* described extensively by Cartan (C 119). Each type of semi-spinors has a quadratic form which is invariant with respect to the group of rotations, namely

$$\xi_0\xi_{1234} - \xi_{23}\xi_{14} - \xi_{31}\xi_{24} - \xi_{12}\xi_{34} \qquad \equiv \phi^T C\phi \qquad \text{for semi-spinors of the first type} \tag{48}$$

$$\xi_1\xi_{234} - \xi_2\xi_{134} - \xi_3\xi_{124} - \xi_4\xi_{123} \qquad \equiv \psi^T C\psi \qquad \text{for semi-spinors of the second type} \tag{49}$$

We follow Cartan: We have three spaces each of eight dimensions, that of vectors, that of semispinors of the first type and that of semi-spinors of the second type, each having a fundamental quadratic form and in which there are three groups of operations which are the same overall , but with two to two correspondences which are not one-one, since to an operation in one of them there correspond two distinct operations of the others. These three groups, considered as acting simultaneously on vectors and on the two types of semi-spinors form a group G which leaves invariant the trilinear form $\mathcal{F} = \phi^T CX\psi$ and the three quadratic forms $F \equiv x^1x^{1'} + x^2x^{2'} + x^3x^{3'} + x^4x^{4'}$,
$\Phi \equiv \phi^T C\phi$ and
$\Psi \equiv \psi^T C\psi$.
The group G can be completed by five other families of linear substitutions which leave the form $\mathcal{F}$ invariant and interchange the three forms F, Φ, Ψ. The operations of each of these new families gives a definite permutation of the three sorts of object, vectors, semi-spinors of the first type and semi-spinors of the second type.

There thus is in the geometry of eight-dimensional, Euclidean space about a point, a principle of triality with three types of objects which play exactly the same role (C120).

We conjecture this triality to be the reason for the three generations observed for elementary particles [19].

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[19] The objections of Zee against a derivation of the three generations from triality in our opinion don't apply. They are based on the group theory of *unitary* transformations which in general have no spin representations (Zee,2016)

3.4. Two ways to trace the physiognomy of complex flat space

3.4.1. The Standard Model: A hybrid of reflections and rotations

The Standard Model of elementary particles is a hybrid of a base layer of spin-1/2-semi-spinors each representing a postulated elementary fermion (see App.B,p.50). These semispinors are embedded in a layer of vector representations of the complex rotations $U(1), SU(2), SU(3)$. Three sets of generators contract into the vectors. The generators play the role of bosons mediating the interaction between the semispinors.

Two characteristic features make the representation based on Cartan distinct from the Standard Model:

(i) the fermions emerge from the space concept instead of being objects postulated externally and added to the space they move in. This means that the Newtonian view that takes matter as external and autonomous against the space it moves in will be discarded.

(ii) reflection operators as natural ingredients of a representation based on spinors take over the role of bosons that the generators of rotations have been acquired in the Standard Model.

3.4.2. Truncated equivalence of rotations and reflections

For QED the use of Dirac gamma-matrices was convention. But when extending to include weak interactions, parity violation for *right-handed* semi-spinors seemed to require another interaction pattern than for *left-handed* semi-spinors. The strategy became to handle all interactions on a semi-spinor base. This meant to use σ- instead of γ-matrices. Both still represented reflection operators. But the Cartan reflection operators σ_i are identical with the generators of $SU(2)$ [20]. With respect to $SU(2)$ we thus find an equivalence between a representation using reflection operators and rotation generators.

A far reaching shift in interpretation appeared when interpreting the σ_i to be generators of $SU(2)$ [21] instead of reflection operators built up from Cartan's $H_i, H_{i'}$.

The formal equivalence of a representation in terms of spin matrices or a representation by generators of rotations ends for $SU(3)$ and higher symmetries. The generators of $SU(n)$ are n^2-1 matrices with dimension n. The elementary objects are thus to be represented by vectors of dimension n which the generators contract into. The dimension of spinors according to the geometrical definition of Cartan but grows with 2^ν. There is no room for a spinor of dimension 3. What for a dimension two appears to be an *isomorphism* between the vector representation of $SU(2)$ and a spinor representation for $\nu=1$ or a semi-spinor representation for $\nu=2$ fades away for higher dimension. There is no straightforward correlation between the Cartan dimension ν and the symmetries $SU(\nu)$.

Generalized gauge invariance then served as a guide to collect $U(1)$, $SU(2)$ and $SU(3)$ under the same roof representing extensions of the translation generator ∂_μ [22]. These rotations determine the apperception of the Standard Model as representing $SU(3)_c \otimes SU(2) \otimes U(1)_Y$. The representation in terms of generators captured the interpretation of the Standard Model with far reaching consequences for the interpretation of objects.

3.4.3. Revitalizing the Newtonian view of matter in space

Although expected to be mostly equivalent because a rotation is equivalent to two reflections a gap exists in interpretation between the Standard Model based mostly on rotations and a concept - we call it SMC - based on flat space defined by reflections.

Leaving reflections and spinors confined to the basement and installing on top a layer in terms of generators of rotations contracting into vectors induced a far-reaching shift in interpretation of the Standard Model. Representing fermions as being

[20] An infinitesimal $SU(2)$-transformation around the identity $S = E - idx^i\sigma_i$ is featuring the generator σ_i embedded into a spin matrix $dx^i\sigma_i$ (Tung 2003,127).

[21] Rotations in 3 dimensions may be written as $= e^{i\frac{\theta}{2}\vec{n}\vec{\sigma}}$ where $\vec{n}$ denotes the rotation axis. The representation of rotations by reflection operators is the unique way to make the law of group multiplication of rotations transparent:

$$S_3S_2S_1 = e^{i\frac{\theta_3}{2}\vec{n_3}\vec{\sigma}}e^{i\frac{\theta_2}{2}\vec{n_2}\vec{\sigma}}e^{i\frac{\theta_3}{2}\vec{n_1}\vec{\sigma}} \tag{50}$$

where $\vec{n_1}, \vec{n_2}\vec{n_3}$ are the rotations axes of three subsequent rotations.

[22] This led to the dominance of $(p=1)$ vector- instead of multivector representations of bosons in the Standard Model.

components of vectors contracting into generators of rotations denies the emergence of particles from the space concept. They become particles that exist *in* space supporting the Newtonian view of matter.

3.4.4. SMC: The objects emerge from the space concept

One of the essential advantages is the emergence of objects from the space concept when using a representation based on reflections. This feature gets lost when the theory gets based on rotations. Then we are dealing with ad hoc objects moving autonomously inside a space reproducing the Newtonian view of matter. Lets have a closer look at the Standard Model:

The Standard Model is featuring a representation in terms of complex rotations (unitary transformations). It is built up by a basic layer of 2-dim semi-spinors each representing a basic quark or lepton. Introduced ad hoc they are grouped into vector components which the generators of $U(1)$, $SU(2)$ and $SU(3)$ are contracting into. This allows the parity violation observed for $SU(2)$ to be accounted for explicitly. The 2-dim semispinors representing the fundamental particles appear to be added ad hoc to become the addressee of the generators of unitary symmetries which as well get postulated ad hoc. Under this predominance of rotations the emergence of objects and of their interactions from the space concept gets lost. The particles as defined in the Standard Model by rotations appear to exist in flat space in complete autonomy independent of this space just as Newton's matter did.

Spinors in the view of Cartan are the parameters that are necessary to span a space by isotropic vectors. These are the natural complex counterpart of the unit vectors in real space. The recursive law that determines the nested construction of their indices perfectly well matches the occurrences of leptons and quarks in powers of two. Moreover: the existence of classes of left- and right-handed particles turns out to reflect the ambiguity of the direction of the normal to a hyperplane in complex space.

The condition of the possibility to measure given by the recursive law that guarantees the spinors to constitute isotropic vectors of length zero when inverted to make manifest the spinor components constitutes the equation of motion. Switching to real space with four dimensions this is the Dirac equation for massless fermions.

The Cartan invariant $\xi^T C \underset{(p)}{X} \xi$ consisting of the product of two spinors with the associated matrix of a multivector provides the equivalent of the Yukawa form of the interaction Hamiltonians postulated in the Standard Model. The experimentally found split into an e.m., a weak and a strong interaction in the SMC is mirrored by representing the Cartan invariant in complex spaces of different dimension ν with the resp. spinors having 2^ν components. The objects in the SMC show the generic sructure imposed by the spinor components: they appear in powers of 2^ν. Indeed we have the electron and the positron (e^+, e^-) as the material base of the e.m. interaction. We have the leptonic and the quark pairs (e, ν) and (u, d) as the material base of weak interactions.

Strong interaction is not as easily conceivable since there is no natural place for (u_r, u_g, u_b), (d_r, d_g, d_b) besides hypothetically appearing as components of the spinor 16^+.

3.4.5. The new role of bosons

There is an important difference between the interaction Hamiltonians of the Standard Model and the Cartan invariants. What are called bosons in the Standard Model are the generators of complex rotations $U(1)$, $SU(2)$, $SU(3)$ that act as ladder operators, whereas in the latter case the associated matrices of multivectors, viz. reflection operators, appear instead. Only for $U(1)$ there is a coincidence of both. But in general the analysis in terms of *rotations* will deliver another pattern as an analysis in terms of *reflections*.

What we might call *bosons* in this mathematical representation are no more the generators of rotations but the associated matrices of p-vectors $\underset{(p)}{X}$, that reside in 2ν-dimensional flat space. The role of ladder operators, viz. creation and annihilation operators that the generators of rotations play in the Standard Model and that essentially determine them to be the mediator of an interaction, in the new reflection determined picture gets overtaken by the reflection operators $H_i, H_{i'}$. They act as the creation and annihilation operators that mediate the transition between the components of spinors.

This means that the calculations and Feynman diagrams no longer can refer to intermediate bosons represented by the generators of $SU(2)$ and $SU(3)$ with all the useful support from generalized gauge invariance. These rotations could be used in the Standard Model because the Lagrangian has been conceptualized as a hybrid consisting of a lower layer describing 2-component spinors introduced arbitrarily for every particle taking part in the game and an upper layer where these spinors are made to become the components of vectors the generators are contracting into. A detailed analysis of the Standard Model is given in app.B, p.50 exposing its hybrid nature consisting of a basement of 2-dim semi-spinors, with a layer on top consisting of vectors defined by rotations.

Our conjecture would mean to replace the hybrid representation of fundamental particles in terms of 2-component spinors addressed by the generators of $SU(2)$ and $SU(3)$ by a representation based on *one* spinor inhabiting the space of dimension ν in which the interaction takes place.

Following Cartan every rotation about an angle θ is equivalent to two reflections on hyperplanes tilted against each other by an angle $\theta/2$. The two representations whence should be equivalent. The practical and theoretical unfolding of the Standard Model made progress along a path driven by the symmetries $SU(2)$ and $SU(3)$. In our opinion there can be no doubt that there is an equivalent path driven by reflections leading to an equivalent experimental verification of a new kind of bosons appearing in the experimental data. [23]

3.4.6. Both Models trace the physiognomy of complex flat space

We conjecture: Bosons may be identified with either the generators of rotations or with p-vectors.

Anyway: what we discover is that the Standard Model with all its ramifications is tracing the physiognomy of a complex flat space characterized by the existence of isotropic vectors and by a set of parameters called spinors guaranteeing this space to be spanned by these isotropic vectors. The representation in different dimensions ν is responsible for the split of the interactions into different types mimicking some equivalent of the e.m., weak and strong interaction.

We know that the Standard Model whose interactions are built up by layers of rotations from general reasons must be compatible with a representation built up by spinors only, because every rotation may be replaced by two reflections.

We thus conjecture that the Standard Model which is based on a layer of rotations should be equivalent to a model - we call it SMC - in which the experimental findings get identified with the respective elements of the spinor structure of flat space as presented by Cartan. Dependend on the theoretical frame bosons experimentally will be identified with either the generators of rotations or with p-vectors.

What we conclude is that the Standard Model with all its ramifications is tracing the physiognomy of a complex flat space characterized by the existence of isotropic vectors and by a set of parameters called spinors guaranteeing this space to be spanned by these isotropic vectors. We conjecture that the representation in different dimensions ν is responsible for the split of the interactions into different types mimicking an e.m., weak and strong interaction.

At this point then our task of proving that the conjecture of Eddington for General Relativity is valid for Elementary Particle physics [24] as well may be taken for granted. Elementary particle physics traces the physiognomy of flat space when this space is taken to be complex and when the elementary particles are identified with the components of spinors.

In case of elementary particle physics the concept of complex flat space as defined by Cartan is encoding the condition of the possibility to measure by providing the new invariant. What we observe experimentally are the Cartan invariants and their intrinsic mechanism of creation and annihilation of spinor components which we metaphorically call fundamental particles and which we erroneously equipp with an autonomous existence in Nature.

The space concept is providing the objects, viz. the fermions, as well as their interactions. Theory and experiment seem to do nothing but tracing the physiognomy of the so defined space. The consistency of the space concept provides the invisible hand that guides experiments to find the inherent structure of this space concept described as to represent fundamental particles. [25]

In each case whether we are using a representation in terms of rotations like the Standard Model or - more refined - in terms of reflections we may conjecture the Standard Model of elementary particle physics to be tracing the physiognomy of a complex flat space concept with all its ramifications.

Elementary particle physics in our opinion is providing the best support for the conjecture of Eddington. There can be no doubt that theory and experiment of elementary particle physics trace the physiognomy of flat space unfolded by Cartan long before any idea of a consistent notion of elementary particles did exist. [26]

For the aim of our paper it is enough to conclude that the Standard Model of elementary particle physics whether exploited

[23] The practical task to construct this representation by reflections will not be achievable without an intense collaboration with the experience of experimentalists as was the case for the development of the $SU(2)$,$SU(3)$ picture favoured in the Standard Model.

[24] at least concerning the successes of QED

[25] We leave it for another task to determine the precise correlation of how the components of the spinors that take part in a specific interaction are related to the generators of rotations.

[26] We should keep in mind that the mathematical concept of a complex flat space represented by reflections has been elaborated by Cartan in 1938, i.e. at a time when not even the slightest notion existed of to what systematics the existence of myons, electrons, protons, neutrons and neutrinos could hint.

by reflections or rotations is tracing the physiognomy of complex flat space.

4. Electromagnetism

4.1. Maxwell's equations trace the antisymmetric bivector of flat space

How identification works in practice most easily can be seen by inspecting the example of Maxwell's electromagnetism:

The century long efforts of Faraday, Ampère and other physicists to obtain a correct theoretical and experimental description of electromagnetic phenomena found its concise summary in Maxwell's eigth equations. They may be grouped into four homogeneous equations:

$$rot\vec{E} = -\partial_t \vec{B} \tag{51}$$

$$div\vec{B} = 0 \tag{52}$$

and four inhomogeneous equations:

$$rot\vec{B} = \partial_t \vec{D} + \vec{j} \tag{53}$$

$$div\vec{E} = \rho \tag{54}$$

These equations traditionally are taken to expose *laws of Nature* that by the efforts of these scientists luckily have been discovered. Eq.(51) is called Faradays law of induction; eq.(52) is stating the law that in the world described by Maxwell's electrodynamics no magnetic monopoles are existing; eq.(53) is called Ampere's circuital law (supplemented by Maxwell's displacement current); eq.(54) states the charge density to be taken as the source of electric fields.

The special relativistic presentation in terms of the bivector $F_{\mu\nu}$ of flat space reveals that the experimentalists were led to choose the *polar* component of the bivector as a reference variable that allowed for an appropriate description of what they measured. Calling it an electric field $\vec{E}$ they soon had to admit that in a complicated fashion it was accompagnied by another field named a magnetic field $\vec{B}$ which showed axial character. What they had detected was the *axial* component of the bivector. The experimental results forced them to notice a strange relation imposed on these two physical fields: the equalness of the square of their amplitudes and their orthogonality. What is a mathematical feature of the polar and axial components of the bivector appeared to be a condition imposed by Nature on the physical fields.

The special relativistic formulation in terms of the antisymmetric bivector $F_{\mu\nu}$ of flat space then leads to the astonishing result, that the *homogeneous* Maxwell equations express nothing but an identity obeyed by the mathematical bivector:

$$\frac{\partial F_{\mu\nu}}{\partial x_\sigma} + \frac{\partial F_{\nu\sigma}}{\partial x_\mu} + \frac{\partial F_{\sigma\mu}}{\partial x_\nu} = 0 \tag{55}$$

Eq.(55) describes an identity as soon as the bivector $F_{\mu\nu}$ is guaranteed to represent an antisymmetric gradient.

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu \tag{56}$$

The *inhomogeneous* Maxwell equations then are seen to express how to identify the measuring entity, the electrical current density J^μ, in terms of the space variables contained in $F_{\mu\nu}$.

$$\partial_\nu F^{\mu\nu} = J^\mu \tag{57}$$

Maxwell's equations in total thus reduce to the requirement

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu \tag{58}$$

stressing the antisymmetric character of the bivector combined with the identification of the measured currents in terms of the bivector

$$\partial_\nu F^{\mu\nu} = J^\mu \tag{59}$$

The requirement $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$ is necessary and sufficient for requiring $F_{\mu\nu}$ to be antisymmetric. This requirement defines $\vec{B}$ to be purely rotational, $\vec{B} = rot\vec{A}$, and hence automatically guarantees $div\vec{B} = 0$. Thus in spite of the fact that theories may be constructed to enforce the existence of magnetic monopoles, the representation in terms of the antisymmetric bivector of flat space is conditioning the non-existence of magnetic monopoles. This is not a law of Nature.

The experimentally as well as theoretically required supplement of Ampere's law, eq.(53), by Maxwell's *displacement current* is a prominent example of how consistency might determine what we will measure.

Maxwell's equations provide a perfect example of how the combined experimental and theoretical efforts of Faraday, Ampere and other famous physicists in effect resulted in tracing the bivector of flat space. What appeared to be *laws of Nature* turned out to describe properties of the bivector. Consistency is not only determining the theoretical picture. It also determines what we are measuring. This way theory will get consistent with observations.

4.2. The quest for a pseudo-euclidean metric

4.2.1. The bivector components to become measurable entities require a pseudo-euclidean metric

The most pronounced example of a mutual conditioning between the space concept and the emergence of objects is given by the bivector of flat space. Till now in all cases we observed the predetermination of the respective objects by the space concept. The electromagnetic field $F^{em}_{\mu\nu}$ offers an example that this is not a one-way street.

It turns out that the wish to identify the polar (x_{14}, x_{24}, x_{34}) and the axial components (x_{23}, x_{31}, x_{12}) of the bivector of flat space with real electric (E_1, E_2, E_3) and magnetic (B_1, B_2, B_3) fields may be satisfied only when spacetime is endowed with a pseudo-euclidean metric. The structure of the space concept hence gets influenced by the way we want to identify the electric and magnetic fields.

This to show we switch to the spinor representation of Cartan. To represent the space of SRT we have to start with $\nu = 2$ and switch to real coordinates. The 4x4-spin-matrix of the bivector then decays into two 2x2-matrices $\underset{(2)}{\Xi}$ and $\underset{(2)}{\Sigma}$ (Cartan 1981,126). Since the bivector is isotropic the determinants of these two matrices have to be zero.

This leads to two constraints on the bivector components which expressed in contravariant coordinates read (Cartan 1981,126):

$$det\ \underset{(2)}{\Xi} = (x^{12} + x^{34})^2 + (x^{31} + x^{24})^2 + (x^{23} + x^{14})^2 = 0 \tag{60}$$

$$det\ \underset{(2)}{\Sigma} = (x^{12} - x^{34})^2 + (x^{31} - x^{24})^2 + (x^{23} - x^{14})^2 = 0 \tag{61}$$

Subtracting the two equations leads to the condition that the polar field x^{14}, x^{24}, x^{34} and the axial field x^{23}, x^{31}, x^{12} have to be orthogonal to each other:

$$x^{14}x^{23} + x^{24}x^{31} + x^{34}x^{12} = 0 \tag{62}$$

which easily gets identified with the familiar restriction

$$\vec{E} \cdot \vec{B} = 0. \tag{63}$$

By adding both equations we but get the *unsatisfiable* condition

$$(x^{23})^2 + (x^{31})^2 + (x^{12})^2 = -[(x^{14})^2 + (x^{24})^2 + (x^{34})^2] \tag{64}$$

translating to $|\vec{B}|^2 = -|\vec{E}|^2$ which to fulfill is impossible.

There s a way to avoid this inconsistency. Spacetime has to be endowed with a pseudo-euclidean metric. By postulating $x^4 \to -icx^4 = -ict$ the bivector components transform to become

$$x^{14} \to -icx^{14}; \qquad x^{24} \to -icx^{24}; \qquad x^{34} \to -icx^{34} \tag{65}$$

leaving us with the constraint

$$c^2((x^{14})^2 + (x^{24})^2 + (x^{34})^2) = (x^{23})^2 + (x^{31})^2 + (x^{12})^2 \tag{66}$$

which then leads to the restriction physicists are familiar with:

$$c^2|\vec{E}|^2 = |\vec{B}|^2\,. \tag{67}$$

Cartan: "*An interpretation of this sort in terms of a real image is possible in the space of special relativity only, but not in real Euclidean four dimensional space.*"(Cartan 1981,132)

The two restrictions eq.(63) and eq.(67) familiar for the electric and magnetic fields hence are rooted in the properties of the bivector of flat space.

4.2.2. Getting a relativistic representation

The electromagnetic tensor $F^{em}_{\mu\nu}$ is defined to be

$$F^{em}_{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{pmatrix} \tag{68}$$

The electric and the magnetic field appear to be the *polar* and *axial* components of this bivector. Under the condition of a pseudo-euclidean metric we are able to identify the experimentally determined electromagnetic tensor $F^{em}_{\mu\nu}$ with the antisymmetric bivector $F_{\mu\nu}$ residing in flat space. For this bivector an identity relation is valid:

$$\frac{\partial F_{\mu\nu}}{\partial x_\sigma} + \frac{\partial F_{\nu\sigma}}{\partial x_\mu} + \frac{\partial F_{\sigma\mu}}{\partial x_\nu} = 0\,. \tag{69}$$

As soon as we require the electromagnetic tensor $F^{em}_{\mu\nu}$ to be antisymmetric by postulating $F^{em}_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$ ($A^\mu = (\Phi/c, \vec{A})$ we may identify the four homogeneous Maxwell equations within eq.(69). Indeed we may replace the four homogeneous Maxwell equations by eq.(69) combined with the additional requirement $F^{em}_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$ ($A^\mu = (\Phi/c, \vec{A})$ to guarantee its antisymmetric character.

The four *inhomogeneous* Maxwell equations in the relativistic notation then get represented by

$$\partial_\nu F^{\mu\nu} = J^\mu \tag{70}$$

saying that the covariant variables of the space concept as compiled in the *bivector* $F_{\mu\nu}$ get identified with the contravariant measuring entities compiled in the relativistic current density $J^\mu = (c\rho, \vec{j})$ [27]:

$$\partial_\nu F^{\mu\nu} = J^\mu \tag{71}$$

The historical efforts of Gauss, Faraday, Ampere and Maxwell hence flowed into tracing flat space by its antisymmetric bivector.

4.3. Electromagnetism and matter

Physicists by checking and organizing experiments in a specific realm of physics are investigating the adopted *space concept* that they don't know yet and even without being aware of what they are doing: highlighting details of this space concept. This space concept provides the *relations* they detect between the experimental outcomes and that guarantees the logical consistency of these relations.

The mathematical bivector experimentally appeared in form of an electric and magnetic field to be identified with its polar and axial components. The electromagnetic field $F_{\mu\nu}$ indeed *is* this bivector whose properties are described by Maxqwell's equations. In Maxwell's equations these fields get coupled to charged currents. The resulting panorama in physical terms is called Maxwell's electromagnetism. It combines the electromagnetic field with a *current* whose nature but appeared obscure. Currents are the way how matter appears on stage. They are made out of electrons. But how to describe these electrons?

4.3.1. Bifurcation: the symmetrical and antisymetrical sector of the space concept

In Coulombs electrostatic theory the electron appeared as a point source with a charge Q the sole pattern of recognition. The underlying space concept, the flat space of classical physics, constituted by the quadratic form that represented invariance against translations and rotations, did not seem to provide any structure to be attached to the electrons. The experiments but detected a till then hidden sector of this space concept opened by the generators of the Lie groups that constituted these classical transformations. Quantum Mechanics for the first time branded the electrons with an existence at its own, identified with wave functions, i.e. the vectors of the Hilbert space spanned by the Eigenfunctions of the generators. The electrons appeared as being a representation of the mathematical support of the generators of translations and Galilei transformations.

[27] taking into account the two restrictions eq.(63) and eq.(67)

The recursive procedure defining spinors when reversed to expose explicitly the spinor components turns out to be the *Dirac equation* now believed to be the equation of motion of the electrons that physicists extracted from their experiments before spinors appeared on stage as a fundamental *antisymmetric* element of the space concept.

Physicists first came into touch with an *antisymmetric sector* of the space concept by the kind of experiments summarized in Maxwell's electromagnetism. Maxwell's theory is based on the mathematical structure provided by the bivector which covers Maxwell's electromagnetic fields coupled to the hitherto unresolved currents by the Cartan invariants.

The antisymmetric derivative of the bivector physically appears as the electromagnetic potential A_μ. When sandwiched with two spinors it provides the *fundamental polar*, a mathematical invariant whose first irreducible component provides for what physicists call the electromagnetic interaction: the two spinors perform as a current density j_μ, which in physical terms *interacts* with the electromagnetic potential.

By the discovery of an *antisymmetric sector* of the space concept containing spinors and the newly detected invariants offered by the irreducible components of the fundamental polar a new horizon got opened announcing elementary particle physics. The theory of spinors unfolded by Cartan became the mathematical base of the experimental results compiled in the Standard Model of Elementary Particle Physics.

The elementary particles appearing in the experiments are to be identified with the components of spinors. Their equation of motion turns out to be the reverse of the defining equation of spinors. Their interactions reflect the irreducible components of the fundamental polar. One of those particles is the electron.

This discovery paved the way to recognize Maxwell's theory as an integral part of elementary particle physics with the electron being the instance of leptons, the e.m. field the instance of an intermediary boson and the e.m. interaction the instance of the Cartan invariant in $\nu = 2$ complex dimensions.

4.3.2. Matter in higher dimensions

The extention of space to higher dimensions gives birth to higher spinor components experimentally identified as leptons and quarks where the electrons were an early example of leptons. In every higher dimensional space the irreducible components of the fundamental polar give rise to new invariants, physically interpreted as new interactions between leptons and quarks in a rough correspondence to the weak and the strong interactions.

This correspondence may not be firmly established before we have evaluated the relation between a representation in terms of generators of rotations $SU(2)$ and $SU(3)$ against a representation in terms of reflection operators.

Finally particles will simply appear as a bump in a probability distribution.

5. Quantum mechanics

The following presentation is taken from the book of Josef M.Jauch, *Foundations of Quantum Mechanics* [28], 1968.

5.1. The condition of the possibility to measure

Jauchs axiomatic approach to Quantum Mechanics is deeply rooted in measure theory. The central question to be answered is: how can a state of a system be measured?

Physically every measurement may be described by a set of yes-no experiments. The results of these experiments constitute a projection-valued spectral measure.

One of the most important problems in measure theory is to identify the class of measurable functions over a measure space (S, M, μ).

If the measure space S is the real line, M are the Borel sets, and μ the Lebesque measure then the integral function $\int d\mu = \Sigma_{i=1}^{n} \alpha_i \mu(A_i)$ is called the Lebesque integral $\int f(x)dx$. Similarly the Lebesque-Stieltjes-Integral is obtained, if we define the measure corresponding to some non-decreasing function $\mu(x)$. We write for this integral $\int_{-\infty}^{+\infty} f d\mu(x)$.

The set of all complex, square integrable functions on a measure space (S, M, μ) is a linear manifold denoted L^2. If we define a scalar product $(f, g) = \int f^* g d\mu$ this manifold is called a Hilbert space. It is the basic mathematical object for Quantum

[28] We try to avoid mathematical subtleties, that tend to obscure the perspecuity of the derivation. Interested readers may consult the original book.

Mechanics, essentially defined by the requirement to allow for measurement.

5.1.1. *yes-no experiments and the propositional calculus*

In order to constitute a physical object physicists have to perform a series of experiments which then in their ensemble will be a full operational equivalent to the constructed *object*. There are properties which depend on the state and there are others which characterize the system and which are therefore independent of the state.

Jauch introduces a particular kind of experiment called *yes-no experiments*; these are observations which permit only one of two alternatives as an answer. Every measurement on a physical system can be reduced, at least in principle, to measurements with a certain number of yes-no experiments (channel analyzer). The yes-no-experiments are referred to as *propositions* of a physical system.

Between certain pairs of propositions there exists a relation which is expressed as e.g. $a \subset b$; meaning: whenever a is true, then b is true, too. What is important for us is that this relation is *independent* of the state of the system. It is the desired structural property which expresses a property of the system independent of its state.

Jauch in axiomatic form presents the structure properties of the propositions of a physical system which are independent of the state. Jauch: *The propositions of a physical system are a complete, orthocomplemented lattice*. The structure of this lattice will be independent of the state of the physical system; the lattice describes the *intrinsic structure* of the system. [29]

5.1.2. *The concept of localizability*

The physical concept of localizability in a natural way leads to a *spectral measure* over the real line Λ.

Taking space represented by Borel sets Δ the propositions locating a particle in various domains Δ of physical space may be represented by a projection-valued measure $\Delta \rightarrow E_\Delta$, that fulfills the following conditions

$$\begin{aligned} E_{\Delta_1} \cap E_{\Delta_2} &= E_{\Delta_1 \cap \Delta_2} \\ E_{\Delta_1} \cup E_{\Delta_2} &= E_{\Delta_1 \cup \Delta_2} \\ \Sigma_{n=1}^{\infty} E_{\Delta_n} &= E_{\cup_{n=1}^{\infty} \Delta_n} \qquad \text{for any seqence of disjoint sets} \quad \Delta_n \\ E_{\Delta'} &= E'_{\Delta} \qquad {}' \text{ denoting the complementary set} \end{aligned} \tag{72}$$

Every spectral measure defines a self-adjoint operator. Stones theorem tells us that every unitary one-parameter group V_β [30] defines a unique spectral measure such that

$$V_\beta = \int_{-\infty}^{+\infty} e^{i\lambda\beta} dE_\lambda \tag{73}$$

The self-adjoint operator defined by the spectral measure eq.(72), the generator of the group, is called the *position operator* Q (Jauch 1968, 197). We may then write eq.(73) equivalently

$$V_\beta = e^{i\beta Q} \tag{74}$$

The one-to-one correspondence of the spectral measures on the real line to self-adjoint operators permits to replace one by the other.

5.1.3. *The canonical commutation relations represent localizability within a homogeneous flat space*

The concept of localizability and the homogeneity of flat space are the key ingredients that constitute the canonical commutation relations. [31]

Two of the basic requirements put on flat space are its *homogeneity* and *isotropy*. Both of these properties express the demand that this space should have no observable physical properties. This means that different points in this space are indistinguishable.

[29] For mathematical subtleties the reader is referred to the Book of Jauch, 1968.

[30] for which $(f, V_\beta g)$ is a continuous function in β for all $f, g \epsilon \mathcal{H}$

[31] In this section we will follow closely Jauch's nomenclature and derivation.

For sake of simplicity let us assume a one-dimensional space. For a particle moving in this space homogeneity may be expressed by requiring that a translation of the space Λ by an arbitrary amount α will induce a *symmetry transformation* in the proposition system (72)(Jauch 1968,197). Following Jauch we introduce the notation $[\Delta]_\alpha$ for the Borel set Δ translated as a whole by the amount α.

Demanding the space Λ to be physically homogeneous means to require that there exist unitary operators U_α such that

$$E_{[\Delta]_\alpha} = U_\alpha^{-1} E_\Delta U_\alpha \tag{75}$$

This relation *"is fundamental. It is the precise mathematical expression of the notion of localizability in a homogeneous space*" (Jauch 1968,198). It constitutes the base of the canonical commutation rules.

The as yet undefined phases of U_α may be chosen in such a way that they form a continuous vector representation of the additive group of real numbers

$$U_\alpha U_\beta = U_{\alpha+\beta} \tag{76}$$

According to Stone's theorem, such a group uniquely determines a self-adjoint operator P:

$$U_\alpha = e^{i\alpha P} \tag{77}$$

P is called the *displacement operator*.

Applying the displacement U_α to eq.(73) we get

$$\begin{aligned} U_\alpha V_\beta U_\alpha^{-1} &= \int_{-\infty}^{+\infty} e^{i\beta\mu} d(U_\alpha E_\mu U_\alpha^{-1} \\ &= \int_{-\infty}^{+\infty} e^{i\beta\mu} d(E_{\mu-\alpha}) \\ &= \int_{-\infty}^{+\infty} e^{i\beta(\mu+\alpha)} d(E_\mu) \\ &= e^{i\alpha\beta} V_\beta \end{aligned} \tag{78}$$

or

$$U_\alpha V_\beta = e^{i\alpha\beta} V_\beta U_\alpha \tag{79}$$

This is the *canonical commutation rule* in *Weyl's* form.

We can throw the canonical commutation rule (79) into still another form by expressing it in terms of the *generators* of the unitary groups.

The generator P is defined on all vectors f for which the limit

$$lim_{\alpha\to 0} \frac{1}{i\alpha}(U_\alpha - I)f = Pf \tag{80}$$

exists, while Q is defined for the vectors f which admit the limit

$$lim_{\beta\to 0} \frac{1}{i\beta}(V_\beta - I)f = Qf \tag{81}$$

From these definitions and eq.(75) we easily obtain the commutation rule [32]

$$[Q,P]f = if \qquad \text{for all } f\epsilon D \tag{82}$$

The notion of *localizability* in a one-dimensional *homogeneous* space in a natural way hence leads to a pair of self-adjoint operators P and Q which on a dense subset D of the entire Hilbert space satisfy the *canonical commutation rule* $[Q,P]f = if$ for all $f\epsilon D$ [33]. Only with these restrictions in mind we can write the formula

$$[Q,P] = iI \tag{83}$$

known as the canonical commutator.

[32] rg: richtig geschrieben, in Gegensatz zu Bohr im Vorspruch.

[33] In order to give this equation a meaning, we must have $Qf\epsilon D_P$ and $Pf\epsilon D_Q$, where D_Q and D_P are the domain of Q and P respectively. The domain D of such vectors f is elsewhere dense and the restriction of Q or P to D is essentially self-adjoint. The spectra of P and of Q are absolutely continuous.

5.2. Switching to dynamics: the Galilean transformation

5.2.1. The evolution in time

In the Schrödinger representation a time displacement operator $U(t,t_0)$ may be defined by $\Psi_t = U(t,t_0)\Psi_{t_0}$ where Ψ_t is the state of the system at time t. Demanding $U(t,t_0)$ to be unitary an infitesimal time displacement reads

$$U(t_0+\delta t, t_0) = 1 - i\,H\delta t \tag{84}$$

where the time displacement operator H has to be hermitian to guarantee U to be unitary. The evolution in time is then described by the Schrödinger equation

$$i\dot{\Psi}_t = H\,\Psi_t \tag{85}$$

Let A be any observable not depending explictly on time. We can define a velocity of A, denoted by $\dot{A}$, by requiring that for any state Ψ_t

$$\frac{d}{dt}(\Psi_t,\,A\,\Psi_t) = (\Psi_0,\,\dot{A}\,\Psi_0) \tag{86}$$

It follows from this definition that for every $\Psi = \Psi_0$

$$i(\Psi,[H,A]\Psi) = (\psi,\dot{A}\Psi) \tag{87}$$

or

$$\dot{A} = i[H,A] \tag{88}$$

In particular choosing for A the position operator Q_r we may define the velocity

$$\dot{Q}_r = i[H,Q_r] \qquad (r=1,2,3) \tag{89}$$

It is only defined if H is known. The $\dot{Q}_r$ thus defined are observables and their expectations values can be measured.

5.2.2. Combining translations and Galilean transformations

If an observer O has measured the observable $\dot{Q}_r$ and has found a value α_r then an observer O' in relative motion with velocity v_r wkill observe the value $\alpha_r + v_r$. The connection between both systems is given by the Galilei-transformation

$$Q_r \to Q_r, \qquad \dot{Q}_r \to \dot{Q}_r + v_r \tag{90}$$

The system is defined to be Galilei-invariant if this transformation is a kinematical symmetry, that is, if there exists a unitary operator G_v which commutes with Q_r and for which

$$G_v\dot{Q}_rG_v^{-1} = \dot{Q}_r + v_r \tag{91}$$

Let us determine the displacement operators H which are admitted by particles satisfying Galilei-invariance. [34]

If we combine the Galilei-transformation eq.(90) with the displacements, we obtain a six-parameter group of translations. We define the family of unitary operators $W(\boldsymbol{\alpha},\boldsymbol{v})$ with the properties

$$Q_r + \alpha_r = W(\boldsymbol{\alpha},\boldsymbol{v})Q_rW^{-1}(\boldsymbol{\alpha},\boldsymbol{v}) \tag{92}$$

$$\dot{Q}_r + v_r = W(\boldsymbol{\alpha},\boldsymbol{v})\dot{Q}_rW^{-1}(\boldsymbol{\alpha},\boldsymbol{v}) \tag{93}$$

It follows from this that the $W(\boldsymbol{\alpha},\boldsymbol{v})$ are a projective representation of the six-dimensional vector space

$$W(\boldsymbol{\alpha_1},\boldsymbol{v_1})W(\boldsymbol{\alpha_2},\boldsymbol{v_2}) = \omega(\boldsymbol{\alpha_1},\boldsymbol{\alpha_2};\boldsymbol{v_1},\boldsymbol{v_2})W(\boldsymbol{\alpha_1}+\boldsymbol{\alpha_2},\boldsymbol{v_1}+\boldsymbol{v_2}) \tag{94}$$

According to the general theory of such representations it is possible to determine the as yet arbitrary phase factors of W in such a way that the factor ω in (94) assumes the form (Jauch 1968,207)

$$\omega(\boldsymbol{\alpha_1},\boldsymbol{\alpha_2};\boldsymbol{v_1},\boldsymbol{v_2}) = e^{i(\mu/2)(\boldsymbol{\alpha_1}\boldsymbol{v_2}-\boldsymbol{\alpha_2}\boldsymbol{v_1})} \tag{95}$$

[34] we do this under the assumption that we are dealing with an elementary system, so that the position operators Q_r generate a maximal abelian algebra.

μ is an arbitrary real constant which is $\neq 0$ and which distinguishes the different inequivalent projective representations of the two-dimensional translation groups.

The two one-parameter subgroups U_α and G_v are recovered by specializing the parameter values according to

$$U_\alpha = W(\boldsymbol{\alpha}, 0) \qquad G_v^{-1} = W(0, \boldsymbol{v}) \tag{96}$$

For these two subgroups, the relation (94) becomes

$$U_\alpha G_v^{-1} = e^{i\mu\boldsymbol{\alpha v}} G_v^{-1} U_\alpha \tag{97}$$

By comparing this with eq.(91) we see that $(1/\mu)P_r$ and $\dot{Q}_r$ have the same commutation rules with G_v. Their difference commutes with G_v; thus it must be a function of the Q_r alone. Hence we find the important relation

$$\mu\dot{Q}_r = P_r - a_r \qquad (r = 1, 2, 3) \tag{98}$$

where $a_r(\boldsymbol{Q})$ are three functions of Q_1, Q_2, Q_3 which may depend explicitly on time.

From the relation (98) we obtain the commutation rules

$$\mu[Q_r, \dot{Q}_s] = i\delta_{rs} \tag{99}$$

We define the operator $H_0 = (\mu/2)\dot{Q}^2$ which then satisfies

$$i[H_0, Q_s] = \dot{Q}_s \tag{100}$$

Consequently, in view of (89) the difference $H - H_0$ commutes with Q_s, hence it must be a function $v(\boldsymbol{Q})$ of the Q_r, which may even depend on time.

The evolution operator H then must have the form

$$H = \frac{1}{2\mu}(\boldsymbol{P} - \boldsymbol{a})^2 + v \tag{101}$$

We may summarize: Every localizable elementary physical system which satisfies Galilean invariance in the sense of (91) evolves in time according to Eq.(85) with H as given by Eq.(101) [35]. This corresponds to the Schrödinger equation in covariant notion.

5.3. The crucial step of identification

We have found that the canonical commutator, $[Q, P] = i\,I$, eq.(83), and the time evolution operator eq.(101) are the direct consequence of the requirement of localizability and homogeneity imprinted on the covariant variables $\boldsymbol{Q}, \boldsymbol{P}, H$).

These relationships are derived entirely within the classical framework of a space concept which is defined by

- *localizability*, expressed by projection-valued spectral measures defined on Borel sets which are equivalent to self-adjoint operators describing the measuring act
- *homogeneity* and *isotropy*, the two basic properties of flat space that express the fact that this space should have no observable physical properties meaning that different points in this space are indistinguishable.
- Galilean invariance and its *combination* with translational invariance [36]

[35] The operators $\boldsymbol{a}(\boldsymbol{Q})$ and $v(\boldsymbol{Q})$ are not entirely determined by Eqs.(98) and (101) since the quantities P_r and H are not determined by their commutation properties. The remaining ambiguity is closely connected with the gauge invariance of the theory.

[36] For the relation of translational and Galilean invariance see Minkowski's talk 1908 on the 80. NATURFORSCHER-VERSAMMLUNG in Cologne at September 21, 1908

> ...Einmal bleibt ihre Form erhalten, wenn man das zugrunde gelegte räumliche Koordinatensystem einer beliebigen Lagenveränderung unterwirft, zweitens, wenn man es in seinem Bewegungszustande verändert, nämlich ihm irgendeine gleichförmige Translation aufprägt; auch spielt der Nullpunkt der Zeit keine Rolle. Man ist gewohnt, die Axiome der Geometrie als erledigt anzusehen, wenn man sich reif für die Axiome

All the derived insights are expressed by the *covariant* variables defining the space concept. To make these insights useful for a viable physical theory the covariant variables defining the space concept are to be brought into relation with the *contravariant* measuring entities $(\vec{p}, E)$ used in the measuring process of classical physics. The insight gained by Eddington says: what happens is that the measuring entities get *identified* with the variables of the space concept just leaving room only for a constant $\hbar$ that balances the different units of covariant and contravariant variables.

To make this procedure transparent we first translate the results we have derived following Jauch's nomenclature to the language of Quantum Mechanics: H and $\boldsymbol{P}$ are referring to the spatial and temporal displacement operators ω and $\vec{k}$ that appear in the exponent of plane waves: $(\omega t - \vec{k}\vec{x})$. The canonical commutator then becomes

$$[q_r, k_s] = i\delta_{rs} \tag{102}$$

and the time evolution operator gets written as

$$\omega = \frac{1}{2\mu}(\boldsymbol{k} - \boldsymbol{a})^2 + v \tag{103}$$

These are the relations that exist between the space variables when localizability, translational and Galilean invariance of the space concept are supposed.

What physicists measure are the contravariant entities $E, \vec{p}, m_0$ in classical mechanics and $V, \vec{A}$ in electrodynamics. What is actually being measured by the physical appliances independent of the names given to them according to Eddington's conjecture are the covariant space variables $\omega, \vec{k},\ \mu,\ v,\ \vec{a}$. The linkage is an *identity* which is hidden by the different unit systems being used. A translation factor $\hbar$ is needed to account for the different units. The identification then reads

$$\begin{aligned} H &= \hbar\omega \\ \boldsymbol{p} &= \hbar\boldsymbol{k} \\ m_0 &= \hbar\mu \\ e/c\boldsymbol{A} &= \hbar\boldsymbol{a} \\ eV &= \hbar v \end{aligned} \tag{104}$$

This translation of the covariant entitities to contravariant entities by means of $\hbar$ directly leads to the Schrödinger picture of Quantum Mechanics. This identification brings the commutator and the Schrödinger equation of Quantum Mechanics to the fore

$$\begin{aligned} [q_s, p_r] &= i\hbar\delta_{sr} \\ H &= \frac{1}{2m_0}(\boldsymbol{p} - e/c\boldsymbol{A})^2 + e\,V \end{aligned} \tag{105}$$

The placeholders $\vec{a}$ and v turn out to refer to the electromagnetic field [37].

The Schrödinger picture is the direct emanation of the translational and Galilein invariance of homogeneous space being represented by means of measurement projectors. In this view Quantum Mechanics is tracing the structure of flat space imprinted by homogeneity and localizability and the requirement of Galilean invariance.

der Mechanik fühlt, und deshalb werden jene zwei Invarianzen wohl selten in einem Atemzuge genannt.
Jede von ihnen bedeutet eine gewisse Gruppe von Transformationen in sich für die Differentialgleichungen der Mechanik. Die Existenz der ersteren Gruppe sieht man als einen fundamentalen Charakter des Raumes an. Die zweite Gruppe straft man am liebsten mit Verachtung, um leichten Sinnes darüber hinwegzukommen, daß man von den physikalischen Erscheinungen her niemals entscheiden kann, ob der als ruhend vorausgesetzte Raum sich nicht am Ende in einer gleichförmigen Translation befindet.
So führen jene zwei Gruppen ein völlig getrenntes Dasein nebeneinander. Ihr gänzlich heterogener Charakter mag davon abgeschreckt haben, sie zu komponieren. Aber gerade die komponierte volle Gruppe als Ganzes gibt uns zu denken auf.
(pdf der wiki Seite, page 4)

[37] It is interesting to note that the principle of Galilean invariance as stated in (91) limits the possible nature of external forces to those of electromagnetic origin. (Jauch 1968,TBD)

5.4. Corollary

5.4.1. The emergence of inertial mass

As a byproduct we get an interesting conjecture concerning the origin of inertial mass. Inertial mass represents the parameter of inequivalence that emerges when translations and Galilean transformations are combined within a 6-dimensional projective translation group. It has the dimension of a reciprocal Compton wave length [38].

Remember that in Jauchs covariant projector derivation of Quantum Mechanics there appeared the parameter μ which eventually would lead to the mass $m = \hbar\mu$. Thus with $m_e = \frac{\hbar}{\lambda_C c}$ we get $\mu = \frac{1}{\lambda_C c}$, i.e. μ must be the reciprocal Compton wave length. The Compton wave length is the de Broglie wave length with $\lambda = h/p$ and with $p = m_e c$ the relativistic momentum of the electron at rest.

Jauch states:

> "The quantity m/μ which connects the displacement operator with the momentum operator is a fundamental constant of the theory, which can be determined by any experiment which relates the measurement of a wavelength (for instance a diffraction) to that of a momentum or energy. This constant is Planck's constant
>
> $$m/\mu = \hbar = \frac{h}{2\pi} \tag{106}$$
>
> . . . $\hbar H$ is the total energy of the particle."
> (Jauch 1968,210) (chapt.12)

In contrast to the objects in the SMC where the fermions are genuinely massless, the objects in Quantum Mechanics genuinely have an *inertial mass* m_0, resulting from including Galilean-invariance, i.e. a dynamical requirement.

5.4.2. The extraordinary role of the superposition principle in Quantum Mechanics

All the fascinating results connected with Quantum Mechanics indicating *entanglement* or such experiments as the *delayed choice experiment* of Wheeler are based on the *superposition* principle. Superposition in the Standard Model but is the feature of an interaction induced by $U(1)$ which is an abelian symmetry. It is not available for interactions characterized by symmetries like $SU(2)$ and $SU(3)$ which are non abelian.

Eddington (1923,149) has shown that $n = 10$ $(\nu = 5)$ is the minimal number of dimensions needed to simulate the effect of a curved space by flat spaces of various dimensions. Only for $\nu = 1$ the attached symmetry $U(1)$ represents an abelian group allowing for superposition.

Superposition hence seems to be a phenomenon that only exists in the lowest dimension and may not be generated by whatever kind of quantization imposed on General Relativity.

Quantum Mechanics is representing a non-relativistic sector of experiments equipped with a superposition principle that stems from the $U(1)$-symmetry characteristic for QED. In contrast to QED its objects are not spinors but wave functions in a Hilbert space. Dirac has shown the inequivalence of spinors with the concept of a Hilbert space. (Dirac 1974)

5.4.3. The condition of the possibility to measure in Quantum Mechanics

The condition of the possibility to measure consists in establishing a *projection-valued spectral measure* on the real line, which admits to define *self-adjoint operators*, namely Q (position) admitting localizability and P (momentum) which admits to define a homogeneous space.

The additional dynamical requirement of Galilean invariance then predetermines the existence of an invariant mass m_0 and fixes a specific Hamiltonian that provides the Schrödinger picture of Quantum Mechanics.

[38] $\frac{\hbar}{m_e c}$ is the Compton wavelength of the electron.

5.4.4. *Quantum Mechanics and SMC: the transition to probabilities*

Since no invariant length may be defined neither in the realm of $QuantumMechanics$ nor of SMC this paves the way for the transition from *measuring* (by comparison with a measuring stick) to *counting* (conditional probabilities). The measurable entity is defined as the relative probability of certain outcomes of scattering experiments.

5.5. The meaning of identification

What is the meaning of identification? According to Eddington identification is a one-sided process: the *contravariant* measuring entities get *identified* with the *covariant* variables defining the space concept, not the other way round.

What is the reason for this one-sidedness? Identification does not mean stating a relation between contravariant and covariant entities with a constant balancing disparate units. Identification in the sense of Eddington means taking for truth that the covariant space variable actually gets the target of the measuring process and it is a matter of habit only, that contravariant units are used. This is the key to the observation that physicists in theory and practice are just tracing the physiognomy of a space concept. They essentially are guided by consistency reasons.

Quantum Mechanics whence is the result of identifying the contravariant measuring entities with the suitably chosen variables of a space concept that allows to encode the condition of the possibility to measure. It's internal consistency guarantees the consistency of the resulting theory with its experimentally determined results.

6. The panorama of physics

6.1. The realms of physics represent distinct epistemes

The access of physics to the world is governed by space concepts. They enable to encode the *condition of the possibility to measure*. Distinct branches of physics - classical mechanics, general relativity, elementary particle physics, quantum mechanics, electrodynamics - are recognized to be distinct because of evaluating distinct aspects of a space concept.

The effort of modern physics beyond classical mechanics consists in unfolding the space concept by either getting rid of restrictions imposed by the classical view as General Relativity did or by relying on flat space but exploiting specific aspects of it not covered before.

- With *Quantum Mechanics* physicists have shifted their focus from the *groups* of symmetry transformations to the Lie algebra of the *generators* of these groups, handling especially translations and Galilei transformations. This enables to get rid of the classical restriction to continuous processes (”natura non fecit saltus”), opening the way to quanta.
- *Electrodynamics* explores the *antisymmetric* structure of multivectors in flat space

From hindsight it seems that the crucial role of physicists consists in eliciting the space concept that allows for the consistency of experimental results and theoretical explanations.

- By turning to *Riemannian space General Relativity* skipped the most severe restriction imposed by using the rigid mathematical objects of Euclidean geometry. The metric tensor believed to be needed for measurement gets identified with the Einstein tensor, the only symmetrical 2-rank tensor derived from the fundamental Riemann-Christoffel tensor. BH's emerge from the metric as well as Keplerian central objects encircled by the geodesics of planets.
- *Elementary particle physics* extends flat space to become a *complex* space equipped with a new mathematical entity called spinor and featuring a new type of invariant that plays the role of an interaction as soon as the spinor components are identified with fundamental particles coming to experimental evidence in the same breath.
- *Quantum Mechanics* (QM) relies on a concept of flat space not defined by mathematical groups of homogeneous symmetry transformations as done in the edifice of classical physics but defined by the *generators* of the Lie algebra equivalent to these transformations supplemented by a position operator defining localizability.

- *Electromagnetism* relies on the asymmetric features of the traditional concept of flat space by identifying the electromagnetic field $F_{\mu\nu}$ with the bivector and the electric and magnetic field with its polar and axial components. The wish to identify these components as real fields becoming object of measurement necessarily requires to equip this space with a pseudo-euclidean metric (see sect.4.2.1, p.23).

6.1.1. Necessary ingredients: an invariant and the condition of the possibility to measure

Two basic features are essential for a space concept to be valuable to become the base for a physical realm:

- the space concept provides for an *invariant* that plays the role of a measuring stick. In case of General Relativity the invariant length element $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ serves this purpose. In elementary particle physics the Cartan invariant of complex space provides for the interaction Lagrangian of QED. In Quantum Mechanics the generators of time and spatial translations provide for the invariant entities energy and momentum.

- the space concept allows to encode a *condition of the possibility to measure*. This condition physically constitutes the basic equation of motion. In General Relativity this is the first Einstein field equation. In elementary particle physics this condition is given by the defining equation of spinors. This mathematical definition, when applied in four dimensions, for fermions physically constitutes the Dirac equation. The bosonic content is defined by the fact that the vectors of this space are supposed to be isotropic which defines the equation of motion of bosons to be $k^2 = 0$.

Consistency is the invisible hand that guides the conceptualization of the experiment and the theoretical representation of the output. The consistency of the space concept will guarantee the consistency of the theory.

6.1.2. The condition of the possibility to measure constitutes the equations of motion

In each realm we find:

The *condition of the possibility to measure* constitutes the basic *equation of motion* that determines physics in this field. This systematic coincidence is the reason for the tremendous power of physics to predict the outcome of measurements. Each successfull measurement necessarily confirms the condition of the possibility to measure.

This condition exposes the *homogeneous* equations, expressed by means of the *covariant* variables of the space concept. The resulting equations are

- in General Relativity the 1. Einstein field equation,

- in elementary particle physics the Dirac equation,

- in Quantum Mechanics the Schrödinger equation,

- in electrodynamics the homogeneous Maxwell equation.

6.1.3. Identifying contravariant measuring entities with covariant space variables

Measurements in most realms of physics have to be performed by using the measuring appliances of classical physics. Physicists are forced to identify the *contravariant* measuring entities of classical mechanics with the *covariant* variables determining the space concept. These identifications constitute the *inhomogeneous* equations. They necessarily must be mediated by a *fundamental constant* which takes into account the different dimensions of the contravariant and covariant entities. The following equations are resulting:

- In case of General Relativity the 2nd Einstein field equation, $G^{\mu\nu} - 1/2g^{\mu\nu}G = -8\pi\kappa T^{\mu\nu}$, which identifies the measuring variables compiled in the energy-stress tensor $T^{\mu\nu}$ $[erg/cm^3]$ with the curvatures compiled in the Einstein tensor $G_{\mu\nu}$ $[cm^{-2}]$. Newton's gravitational constant κ $[cm/g]$ plays the role of a mediator

- in case of Quantum Mechanics this identification is expressed by the basic postulate [39] $p_\mu = \hbar k_\mu$ with [40] p^μ $[erg]$ representing the contravariant energy-momentum vector of particles, k_μ $[s^{-1}]$ the covariant wave vector of plane waves and Plancks action quantum $\hbar$ $[erg/s^{-1}]$ taking the role of the mediator;
- in case of *electrodynamics* the inhomogeneous Maxwell equations, $F^{\mu\nu}_{\nu} = J^\mu$, identify the contravariant measuring variables contained in the electric current density J^μ with the covariant variables compiled in the divergence of the electromagnetic field $F^{\mu\nu}_{\nu}$. The electric charge e $[(gcm)^{1/2}]$ plays the role of the mediator. The charge does not appear explicitly in this identification since by historical reasons it has been integrated in the definition of the electromagnetic field, which then acquired a dimension $[charge/cm^2]$.

6.1.4. Properties of physical objects are mirroring features of the underlying space

- The defining equation for spinors in 4-dim space physically reappears to be the Dirac equation. The complicated structure of their indices mathematically induced by their defining recursion formula gets mirrored in the structure that governs the occurrence of the elementary particles, the leptons and quarks. A new constant appearing in spinor space in 4 dimensions physically reappears to be the interaction Hamiltonian of QED with its characteristic Yukawa shape.
- a mathematical property shown by the generators of space translations when combined with Galilei transformations physically reappears as the Schrödinger equation and the characteristic commutator of Quantum Mechanics as soon as the experimental entities compiled in p^μ get identified with the space variables, compiled in the wave vector k_μ.
- Maxwell's homogeneous equations appear to be an identity of the mathematical bivector in flat space. A mathematical property of the polar and axial components of the bivector demands the conditions electric and magnetic fields are subjected to in Maxwell's theory, viz the equalness of their absolute value and their characteristic orthogonality.

6.2. Objects as well as their interactions emerge from the condition of the possibility to measure

To give the equations of motion a physical meaning an object has to be physically postulated and has to be identified with its mathematical counterpart in this equation. This is an aspect of *identification* not expressed by mathematical equations. It constitutes a fundamental step in erecting the physical edifice. The objects emerge as soon as the condition of the possibility to measure gets specified in the respective realm of physics.

The existence of objects and the existence of their interactions that make them recognizable by the physical apparatus are mutually conditioning each other. Both, the objects of the theory as well as their interactions, are an emergent feature of the space concept.

- In *General Relativity* the astronomical objects emerge from the *condition of the possibility to measure*, i.e. from the 1^{st} Einstein field equation, as soon as the invariant length ds^2 gets restricted to respect some additional conditions: besides the requirement on the solution to be static (to be solvable at all) and to display spatial spherical symmetry (to allow for an isolated object) and to be invariant against time inversion it should approach the Minkowski metric at $r \to \infty$ (since no influence from any mass is expected there). The existence of a Lorentzian metric is the most basic requirement on General Relativity to assure the approximate validity of SRT in spacetime regions which are small compared to the time and distance scale set by the curvature of spacetime (Ehlers 2007,92). Under these conditions the Schwarzschild metric appears. The existence of objects is signaled by a factor $(1 - 2m/r)$ in the temporal and radial part of the metric $g_{\mu\nu}$ announcing the singularity structure of a massive black hole (BH) with its covariant mass m given in $[cm]$.

  The interaction of these objects is physically determined by identifying the metric $g_{\mu\nu}$, from which the object is emerging, with a physically acting gravito-inertial field. [41]

  Calculating the geodesics [42] by means of this metric then produces all the successes of early General Relativity - the *perihelion shift of Mercury*, the *deflection of light* and the *redshift of light* in a gravitational field.

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[39] Quantum Mechanics is a non-relativistic theory. It is only as a convenient shorthand notation that we are using a relativistic nomenclature

[40] we use units with the convention $c = 1$

[41] Be aware of Eddington (1975,221,fn.*): *"An electromagnetic field is a "thing"; a gravitational field is not, Einstein's theory having shown that it is nothing more than the manifestation of the metric."*. This differentiation makes it possible to speak of empty space in contrast to space filled with matter.

[42] The existence of the geodesics is guaranteed as a consequence of the energy-stress tensor being divergence free, which by construction is automatically fulfilled (see next section)

- in elementary particle physics fundamental fermions, the leptons and quarks, physically get postulated and identified with the components of spinors ξ_α. These physical objects by being identified with *isotropic* mathematical structures necessarily are *massless* thereby reproducing the maslessness of the fundamental fermions constituting the Standard Model before the advent of the Higgs field.

  Inverting the *defining equation* of spinors that describes how to span the complex flat space by isotropic vectors results in the fundamental *equation of motion* $P\xi = 0$ with P the associated matrix of the vectors that span complex flat space. This may be seen by specializing to complex dimension $\nu = 2$ corresponding to dimension $n = 4$ in real space: we find the *Dirac equation* $\not{p}\xi = 0$ for a massless fermion.

  The transition of flat space to complex makes appear the new Cartan invariant that allows to become identified as an *interaction*.This identification of the invariant is possible as soon as its ingredients, the *spinors*, get identified with a set of fundamental *fermions*, accompanied by the identification of *bosons*, called the mediators of the interaction, as the associated matrix of the multivector involved in the invariant.

  The Cartan invariant in the dimension $\nu = 2$ turns out to be identical with the interaction Hamiltonian of QED [43]. In higher dimensions the invariant reproduces another set of interactions acting between the fermions that get identified with the spinor components available in this dimension.

- In electromagnetism the electromagnetic field got postulated by Maxwell and identified with the bivector $F_{\mu\nu}$ of flat space. The electric and magnetic fields, $\vec{E}$ and $\vec{B}$, experimentally described by Faraday, Gauss and Ampère, turned out to be the polar and axial components of the bivector automatically obyeying the two relations characteristic for the respective components: $\vec{E} \cdot \vec{B} = 0$ and $|\vec{E}|^2 = |\vec{B}|^2$. To achieve this identification Maxwell saw himself forced to supplement the configuration by postulating a *displacement current*.

  A complete description of the interaction of the electromagnetic field $F_{\mu\nu}$ with matter is only addressable in the framework of the spinor theory of elementary particles.

7. General Relativity and Quantum Mechanics: A deep gulf and a structural affinity

7.1. The deep gulf between General Relativity and Quantum Mechanics

There is a deep gulf separating Quantum Mechanics from General Relativity. General Relativity and Quantum Mechanics have a distinct *epistemological* foundation.

General Relativity is founded on Riemannian space and sticking on a representation by tensors. According to Norton (1993) as a result of eight decades of debate the trademark of General Relativity is the denial of the existence of any absolute object that is acting but is not acted upon. This excludes any definition of space by transformations, because these automatically would require the existence of an absolute object.

The operational space of Quantum Mechanics in contrast is spanned by the generators of a Lie algebra referring to translations combined with Galilei transformations. Moreover it necessarily refers to the pre-relativistic separation of space and time which not even allows for a spacetime metric to be defined.

Whereas General Relativity heavily relies on the definition of a metric which turns out to be the gravito-inertial field Quantum Mechanics from fundamental reasons doesn't know of any metric. Quantum Mechanics is necessarily non-relativistic since the imaginary character of time in a pseudo-euclidean metric does not allow to associate time with a hermitian operator which would be necessary to make time a measureable observable.

Quantum Mechanics shows a quantization based on the Planck constant $\hbar$ which has become its hallmark. This feature is a direct consequence of the specific way the identification of contravariant mesuring entities $(E, \vec{\boldsymbol{p}})$ with the covariant space variables $(\omega, \vec{\boldsymbol{k}})$ has been conceptualized in Quantum Mechanics. The commutator between space and momentum characteristic for Quantum Mechanics is the 1:1 translation of the fundamental commutator between location and its displacement.

Comparison with the Standard Model indicates that the Schrödinger equation covers the e.m. interaction defined by a $U(1)$ symmetry leading to the superposition phenomena that have become the second hallmark of Quantum Mechanics.

[43] referring to the original formulation of QED using the Dirac matrices γ_μ as 4-dim representation of the Clifford-algebra

7.2. The cousinship of General Relativity and Quantum Mechanics

But nevertheless there also is a deep structural affinity between both. General Relativity and Quantum Mechanics both give an answer to the same question: how can a measurement take place under the condition that no material mesuring stick is available? This is the problem for measurements *below the distance of atoms* (Quantum Mechanics) and for measurents in the *empty universe* (General Relativity). The answers look very different but in both cases rely on the introduction of a *covariant* entity taking the role of a measuring stick: the *wave vector* k_μ in case of Quantum Mechanics and the local directed *curvatures* [44] contained in $G_{\mu\nu}$ in case of General Relativity.

7.2.1. Theater of identification set by kinetic energy (Quantum Mechanics) or by mass (General Relativity)

To make them a physical theory both concepts need to identify the *contravariant* measuring entities of classical physics with the *covariant* variables of the resp. space concept.

In Quantum Mechanics the contravariant measuring entities compiled in the energy-momentum vector p^μ [45] get identified with the displacement vector k_μ specifying the space concept:

$$p^\mu = \hbar k^\mu \tag{107}$$

The contravariant energy is measured in $[erg]$. The equivalent covariant wave entity, the frequency, is measured in $[s^{-1}]$. The Planck constant therefore has the dimension $[erg \cdot s]$.

In General Relativity the contravariant measuring entities compiled in the energy-stress-tensor $T^{\mu\nu}$ get identified with the covariant curvatures contained in $G_{\mu\nu}$ by demanding

$$-8\pi\kappa T^{\mu\nu} = G^{\mu\nu} - 1/2 g^{\mu\nu} G \qquad (2^{nd} \text{ Einstein field equation}). \tag{108}$$

The *heavy mass* as a representative of the space concept appears on the right hand side, equipped with a dimension $[cm]$ (see Eddington 1975,p.85) . Its equivalent in the classical measuring arena, the inertial mass, is appearing on the left hand side of eq.(108) with the dimension $[g]$ (see Eddington 1975,116,130). The Newtonian gravitational constant κ as the mediator between covariant and contravariant entities hence has the dimension $[cm]/[g]$.

The systematic difference between both constants, $\hbar$ and κ, hence reduces to the fact that Quantum Mechanics is featuring the transition between contravariant and covariant entities on a stage set by the *kinetic energy* whereas General Relativity is featuring this transition on a stage set by the *mass*.

Hence besides the deep gulf separating Quantum Mechanics and General Relativity there as well does exist a deep structural affinity of Quantum Mechanics and General Relativity.

The founding equation of Quantum Mechanics, eq.(107), is the 1-dimensional analogue of the 2-dimensional founding equation of General Relativity, eq.(108), Einstein's 2^{nd} field equation.

General Relativity whence is performing the same step of identification as Quantum Mechanics, but in a 2-dim tensor world instead of a 1-dim vector world.

The Planck constant $\hbar$ epistemologically plays the same role in Quantum Mechanics as does Newton's gravitational constant κ in General Relativity.

7.2.2. Geodesics and dispersion relation

The results that made early General Relativity famous were reached without Newton's gravitational constant κ intervening.

But an additional assumption was necessary fixing the orbit of an object to be a *geodesics*

$$\frac{d^2x^\mu}{ds^2} - \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0 \tag{109}$$

This assumption allowed to correctly calculate the *Perihelion shift* of Mercury and the *deflection of light* in the gravitational field of the sun.

[44] the local directed curvatures, see the detailed specification in Eddington 1975

[45] written in special relativistic notation in spite of the fact that Quantum Mechanics does not admit a special relativistic form

In Quantum Mechanics a similar situation exists. The results for e.g. the large sector of Quantum Optics are got without Plancks constant $\hbar$ intervening. Also in this case an additional postulate had to be introduced, the requirement of the existence of a *dispersion relation*

$$\lambda \cdot \nu = c \tag{110}$$

It guarantees the existence of light rays proceeding by means of a wave front perpendicular to the displacement vector k_{μ}. This assumption then allows for the correct calculation of e.g. the delayed choice quantum eraser [46]. The additional postulate whence serves to fix the geometry not by a mathematical postulate but by means of a physical object [47]. The dispersion relation allows for the transition from the covariant units $[s^{-1}]$ of the frequency to the contravariant units $[cm]$ of wave length.

In both cases be it the geodesics in a curved world or the dispersion relation in flat space the additional requirement defines the shortest or longest distance between two positions. The path of a physical object, a planet in case of General Relativity, a light ray in case of Quantum Mechanics, is used to specify the geometry.

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7.2.3. *Identification is the entrance door for the logical figure of mutual conditioning*

Quantum Mechanics has become known for the mutual conditioning of waves and particles. Particles may behave as waves and waves may show features of a particle. This mutual conditioning results from the identification of the classical contravariant measuring entities E and $\vec{p}$ with the covariant frequency ω and the wave vector $\vec{k}$.

In General Relativity a similar phenomenon may be observed, the mutual conditioning of matter and space - the distribution of matter determines the metric, the metric determines where matter flows.The mutual conditioning of matter and space results from the identification of the energy-stress tensor $T^{\mu\nu}$ with the Einstein tensor $G_{\mu\nu}$.

8. QED in spite of its name is not a quantum theory

A careful inspection (see app.C, p.53) reveals that no result of QED displays any dependence on $\hbar$ despite the fact that the Feynman rules are interspersed with occurrences of $\hbar$. The only dependance the results of QED show is on the fine structure constant $\alpha_{em} \approx 1/137$, the coupling constant of the e.m. interaction. By convention only QED is embedded into a mathematical enveloppe making the Feynman rules display $\hbar$.

The effect of quantization is produced by the appearance of creation and annihilation operators in the scattering amplitudes. In QED they are introduced by the basic commutator $[q, p] = i\hbar$ of Quantum Mechanics. The representation based on the spinors of Cartan but shows that they reflect the effect of reflection operators when acting on the components of spinors.

8.1. The role of the commutator $[q, p] = i\hbar$

The canonical commutator $[q, p] = i\hbar$ of Quantum Mechanics has a classical origin. It results from the purely classical so called Weyl commutator $[Q, K] = i$ for the generator K of translations and the position operator Q [48].

The special consequences of these purely classical results induced in Quantum Mechanics result as soon as the measuring entity $\vec{p}$ gets identified with the generator $\vec{k}$ by setting $\vec{p} = \hbar\vec{k}$. This identification and its temporal equivalent $E = \hbar\omega$ give Quantum Mechanics its special and fascinating role.

The canonical commutator is easily shown to be equivalent to the commutator of creation and annihilation operators for bosons $[a, a^{\dagger}]_{-} = 1$. [49] This transition first shown for the harmonic oszillator finds wide application in the procedure of

[46] Gaasbeek, Bram, Demystifying the Delayed Choice Experiments, arXiv:1007.3977v1 [quant-ph]

[47] The technique of freqency combs has allowed to exploit an ever broader regime (s. BPM TBD)

[48] This commutator simply reflects the classical commutator $[\partial x, x] = 1$ which embodies the classical multiplicative law of differentiation.

[49] By setting

$$a^{\dagger} = \frac{1}{\sqrt{2}}(\frac{p}{\hbar} + iq), \qquad a = \frac{1}{\sqrt{2}}(\frac{p}{\hbar} - iq) \tag{111}$$

we easily from $[q, p] = i\hbar$ get the relation:

$$[a, a^{\dagger}]_{-} = 1 \tag{112}$$

which is independent on $\hbar$. It shows the signature of creation and annihilation operators of bosons.

second quantization, which marks the transition to occupation numbers in Fock space and hence the entry to QED. This transition is getting rid of the Planck quantum $\hbar$.

8.2. The reflection operators acting on spinors as creation/annihilation operators suggest QED to be a quantum theory

This canonical commutator of Quantum Mechanics is the historical origin for the appearance of creation and annihilation operators in the amplitudes of QED which are responsible for the quantization effects. But there is a deeper reason for this appearance.

In a representation based on Cartan the reflection operators are the backbone of the representation of kinematic variables as associated matrices. They first appeared as γ-matrices, $\gamma_\mu p^\mu$, in the Dirac equation and as $\tilde{\psi}\gamma_\mu A^\mu \psi$ in the interaction Hamiltonian of QED. These reflection operators when acting on spinor components are acting as creation and annihilation operators (see $3.2.2$, p.14).

The Cartan induced representation based on spinors and reflection operators demonstrates that the commutator of Quantum Mechanics does not play any role in QED. The impression of quantization is effected by the appearance of creation and annihilation operators offered by the reflection operators when acting on spinor components.

The fundamental constant $\hbar$ hence does not play any role in this mimicking of quantization. $\hbar$ is not an indicator for a quantization taking place in QED. And indeed no result of QED displays any dependence on $\hbar$.

8.3. The quest for quantizing General Relativity

General Relativity till now is resisting all attempts to apply known quantization procedures of quantum field theory. We are urged to ask another question: what are the reasons responsible for the insistence with which quantization of General Relativity is demanded for?

One reason seems to be the success of QED which suggests that a field theory has to be quantized to become successful. Another one might be the wish to integrate such fascinating phenomena like entanglement or the delayed choice experiment of Wheeler into General Relativity. And another one the misinterpretation the canonical commutator of Quantum Mechanics could suggest a quantization of spacetime itself and hence of General Relativity as the all encompassing theory of spacetime.

We have shown why the success of QED does not demand for quantizing General Relativity. In spite of its name QED is not a quantum theory. The results of QED do not show any dependence on $\hbar$ but are dependent on the e.m. coupling constant α_{em} only (see sect.($8.3.2$)).

The canonical commutator of Quantum Mechanics has its roots in the classical commutator between the location and the displacement operator. It does not invite to such far reaching conclusions as a quantization of General Relativity.

And we will show why phenomena based on the superposition principle of Quantum Mechanics are not plausible to reflect some basic feature of General Relativity.

Instead we have to reevaluate the relation between Quantum Mechanics and General Relativity. This reevaluation shows that General Relativity already possesses all the features that make it a fair representative of Quantum Mechanics in the general relativistic realm.

8.3.1. The success of QED does not provide any reason for quantizing General Relativity

Quantum electrodynamics (QED) historically marks the attempt to develop Quantum Mechanics to become a relativistic field theory. It has become the most successfull theory of physics. A deeper analysis but shows: QED though historically descendent from Quantum Mechanics and in spite of its naming genuinely has nothing to do with Plancks action quantum $\hbar$.

8.3.2. QED does not suggest General Relativity to become quantized

We conclude that the familiar view that QED be the relativistic encoding of a quantum theory based on Planck's constant $\hbar$ is misleading. The success of QED seduces to erroneously demand field theories to become quantized. This seems to be the origin of the insistence of the quest for quantizing General Relativity.

Part of the magics of quantization seems to derive from its alleged role in the fascinating and successful field of elementary particle physics. In our view this is a fallacy. We have shown that QED is tracing the physiognomy of a complex flat space represented by reflections. Nowhere in this theory does $\hbar$ play any essential role.

The commutator $[q,p] = i\hbar$ eventually has been taken to suggest a quantization of space and hence, since General Relativity is a theory of space, to suggest a quantization of General Relativity. The origin of this commutator as derived above but suggests there to be no hint to any quantization of space in this commutator.

8.4. The new type of fundamental constant α_{em} in QED

In elementary particle physics the fundamental particles, the leptons and quarks, are identified with the covariant variables of the space concept, the spinor components. It is the Cartan invariant which gets identified with the interactions of these objects. No special identification of contravariant measuring entities with covariant space variables has to be performed.

This is in striking contrast to the case of General Relativity - requiring κ - and to the case of Quantum Mechanics - requiring $\hbar$ for identification of the measuring entities with the variables of the space concept.

This is the reason why the Planck constant $\hbar$ cannot play any other than spurious role in elementary particle physics as compiled in QED.

If we follow the conjecture of Wyler (1968) the coupling constant of the e.m. interaction α_{em} - like the coupling constants of the weak α_{weak} and of the strong α_{strong} interactions (TBD) - are determined completely by the structure of flat space, viz. the ratio of group volumina as determined for flat space by Hua (TBD) [50].

In striking contrast to the Newtonian gravitational constant κ and Planck's constant $\hbar$ these coupling constants hence are not constants fed in from the outside but are representing intrinsic features of the higher dimensions of flat space.

9. The location of elementary particles in General Relativity

A theory of matter based on Cartan being a theory of *isotropic*, i.e. vectors of length zero, then suggests its location within the framework of general relativity.

9.1. Antisymmetric features within the space concept

Eddington already in 1923 conjectured the e.m. field $F_{\mu\nu}$ to be the antisymmetric part of the contraction $G_{\mu\nu}$ of the Riemann-Christoffel tensor:

$$G_{\mu\nu} = B^{\lambda}_{\mu\nu\lambda} \tag{113}$$

We write $G_{\mu\nu} = R_{\mu\nu} + F_{\mu\nu}, \;\; R_{\mu\nu} = 1/2(G_{\mu\nu} + G_{\nu\mu}), \;\; F_{\mu\nu} = 1/2(G_{\mu\nu} - G_{\nu\mu})$. Identifying the metric tensor with a multiple of the redefined Einstein tensor

$$ds^2 = \lambda G_{\mu\nu} dx^{\mu} dx^{\nu} \tag{114}$$

leads to

$$ds^2 = \lambda R_{\mu\nu} dx^{\mu} dx^{\nu} \tag{115}$$

The contribution of the *antisymmetric* part of $G_{\mu\nu}$ to the invariant length element is zero. This is the sector in which fermions, elementary particles, which are defined by being isotropic, i.e of length zero, are located [51].

Till now we could state that space and matter are conditioning each other: the distribution of matter is determining the space curvature that determines how matter gets distributed. With the inclusion of an antisymmetric aspect of matter this statement remains true but needs refinement. Matter and space now are much more intermingled than suggested by the traditional picture. Both now split up to show symmetric and antisymmetric aspects that physically are clearly distinct.

[50] Their derivation on purely group theoretical grounds did not know the Cartan framework into which we have embedded the theory of elementary particles.

[51] We note that the traditional requirement that the Riemann-Christoffel tensor $B^{\lambda}_{\mu\nu\rho}$ should be symmetric in μ and ν in the sense of Anderson (Anderson, Gautreau 1969,1657) would mean to postulate an absolute object not admitted in General Relativity.

9.1.1. *Eddington incorporates the e.m. field*

Eddington paved the way by locating the e.m. field $F_{\mu\nu}$ to be the antisymmetric part of the contraction of the Riemann-Christoffel tensor whose symmetric part is constituting the metric tensor $g_{\mu\nu}$ to become identified with the gravito-inertial field.

Eddington had to restrict himself to the implementation of the electromagnetic field into the frame of General Relativity. No viable theory of matter did exist at his time. Replacing the reference to the mathematical procedure of *parallel displacement* by a *principle of identification* allowed him to circumvent (Edd 1923,222) the criticism of Einstein and others regarding a similar bold venture undertaken by Weyl.

Instead of deriving the e.m. field by parallel displacement Eddington *identified* the e.m. field with the antisymmetric part of the contraction of the Riemann-Christoffel tensor. This implied that the symmetric Einstein tensor $G_{\mu\nu}$ would acquire an antisymmetric counterpart $F_{\mu\nu}$. Fed into the invariant length definition $ds^2 = G_{\mu\nu}dx^\mu dx^\nu$ this antisymmetric contribution would lead to $ds^2 = 0$. This indeed is the trajectory of light in General Relativity.

$F_{\mu\nu}$ automatically is obeying the homogeneous Maxwell equations, since [52] it may be derived from a potential $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The coupling of this field to matter phenomenologically is described by the inhomogeneous Maxwell equations $\partial_\nu F^{\mu\nu} = J^\mu$ with J^μ an e.m. current density [53]. This is the way the contravariant measuring entities of classical physics sampled in the e.m. current density get identified with the space concept variables, viz. the bivector inhabiting flat space. The Maxwellian theory of electromagnetism hence easily derives from analyzing the behaviour of the bivector of flat space.

9.1.2. *The disappearance of elementary particles in the formalism of General Relativity*

The symmetric part generates the nonvanishing contribution to the invariant-length element ds^2. The contribution of the antisymmetric part to ds^2 is zero. The antisymmetric nature of the e.m. field $F_{\mu\nu}$ leads to the square of the photon momentum being zero, $k^2 = 0$. But this is the genuine definition of *fermionic* matter characterized by reference to objects of length zero.

Fermionic matter is representing the *antisymmetric* properties of the space concept while gravitation is representing the *symmetric* properties. The bosonic fields $F_{\mu\nu}$ mediating the interaction of the matter particles appear to be the antisymmetric twin of the symmetric metric tensor $g_{\mu\nu}$ which represents the gravitational interaction. The e.m. field which is part of the theory of elementary particles finds its position at the location Eddington devised to it (Edd 1923,223), since it by its genuine nature represents $ds^2 = 0$.

This kind of matter hence necessarily does not appear explicitly in a theory like General Relativity whose calculations are concentrated around the invariant length element. General Relativity puts its emphasis on objects with a geodesics decribed by $ds^2 \neq 0$

9.2. Symmetric and antisymmetric matter

Before elementary particle physics entered the stage we found an explicit mathematical representation of the *symmetric* appearance of matter. The gravito-inertial field $g_{\mu\nu}$ made its appearance in the invariant length element and determined the interaction with other matter. What it was but that interacted - namely the BH or the Keplerian central object or even the planet Mercury - had to be inferred by some respective disturbance of the symmetric metric tensor, which only in case of the BH showed a clear mathematical signature expressed by the factor $(1 - 2m/r)$. We might talk about *symmetric* matter which with the appearance of elementary particles got an *antisymmetric* counterpart.

The interactions of the *antisymmetric matter* gets mediated by the *associated matrices* of antisymmetric multivectors of flat space. The associated matrices are rooted deeply in the spinorial representation of space. An example of an associated matrix within the four dimensions of SRT is given by $\not{A}$, the Dirac slash representing the product $\gamma^\mu A_\mu$. Each hint to the spin structure but gets lost at the surface that General Relativity presents by founding its physical horizon on the invariant length element.

Eddington succeeded to identify the General Relativity location of the electromagnetic field $F_{\mu\nu}$ as the antisymmetric part of the contraction of the Riemann-Christoffel tensor. This assured the validity of the homogeneous Maxwell equations. But

[52] as Eddington (1923,219) showed under very general conditions

[53] Eddington was confronted with the difficulty of how to identify this charged matter current density as long as no microscopic theory of matter did exist. This set a halt to his aim to deduce theoretical physics from a space concept by using his principle of identification.

without any spinorial representation this field was devoid of any deeper relation to matter. It could be formally coupled to the current density of electrodynamics

$$\partial_\nu F^{\mu\nu} = J^\mu \tag{116}$$

but without any microphysical representation of the current density J^μ being available.

Antisymmetric matter is identified with spinors. Their components are identified to be the fundamental particles that take part in interactions that depend on the dimension of flat space. What makes up '*that what interacts*' is clearly defined. And also this interaction is clearly defined: spinors are constituting the *Cartan invariant*, an interaction term that is completely independent of the invariant length element used in General Relativity. What constitutes *antisymmetric* matter and its interaction is much clearer defined as what means *symmetric* matter.

Based on the role of isotropic vectors in Cartan's space concept we would be well advised to conjecture antisymmetric matter to be located in the antisymmetric part of General Relativity characterized by an antisymmetric equivalent of the Einstein tensor leading to isotropic phenomena with $ds^2 = 0$. The antisymmetric part is comprising bosonic fields like the e.m. field, $F_{\mu\nu}$, while the symmetric counterpart being represented by the gravito-inertial field $g_{\mu\nu}$.

9.3. Conditional existence

To speak of the location of elementary particles in General Relativity but is misconceiving the gulf that separates a representation by tensors as used in General Relativity and a representation by spinors as required by the concept of a complex flat space. The problem does not derive from the space being complex but from the problem that spinors "*have metric but not affine characteristics.*"(Cartan 1938,Introduction) [54]). To have "*affine characteristics*", i.e. to be valid in any coordinate system whatsoever, is a prerequisite to become accepted in General Relativity. Cartan claims it to be impossible to handle spinors in the Riemannian space with the usual techniques [55].

Postulating the existence of spinors in Riemannian space leads to difficulties which according to Cartan are *insurmountable* with the techniques usually applied in Riemannian space. But the existence of elementary particles irrevocably seems to be attached to the existence of spinors. The existence of elementary particles thus seems to be bound to the possibility to consider the respective space to be flat. What to do?

9.3.1. Permanent existence vs. conditional existence

There might be a deeper reason that prevents the explicit appearance of fermionic matter in General Relativity. The theory of General Relativity is deemed to be generally covariant. Whereas every theory, even Newton's theory, might be represented by a general covariant formalism, General Relativity is different by not admitting so called absolute objects (Anderson, Gautreau 1969,1657). Those are objects that act but are not acted upon, like the Minkowski metric in SRT or the absolute filiation of space and time in Newton's theory. General Relativity is based on affine objects, i.e. affine tensors that persist against whatever homogeneous transformations. Spinors are tensors against rotations and reflections but they are not affine. Cartan insists on a purely geometrical origin of spinors. This origin makes it easy to introduce spinors into Riemannian geometry and particularly to apply the idea of parallel transport to these geometrical entities. But:

> "the difficulties - difficulties which are *insurmountable* (emphasis by Cartan) if classical techniques of Riemannian geometry are used - can be explained. These classical techniques are applicable to vectors and to ordinary tensors, which, besides their metric character, possess a purely affine character; but they cannot be applied to spinors which have metric but not affine characteristics" (Cartan 1966,Introduction) [56].

[54] "Finally this geometrical origin makes it very easy to introduce spinors into Riemannian geometry, and particularly to apply the idea of parallel transport to these geometrical entities. The difficulties which have been encountered in this respect - difficulties which are *insurmountable* (emphasis by E.Cartan) if classical techniques of Riemannian geometry are used - can be explained. These classical techniques are applicable to vectors and ordinary tensors, which, besides their metric character, possess a purely affine character; but they cannot be applied to spinors which have metric but not affine characteristics."

[55] Attempts (e.g. L.Infeld and B.L.van der Waerden) to avoid this impossibility - by decoupling the spinor transformation properties from their geometrical origin - were rejected by Cartan as "geometrically and even physically so startling" (french: "choquant") (Cartan 1938,151)

[56] Cartan makes this impossibility be subject of a fundamental theorem:

> "*...; that is having chosen an arbitrary system of coordinates x^i for the space, it is impossible to represent a spinor by any finite number N whatsoever of components u_α such that the u_α have covariant derivatives of the form $u_{\alpha,i} = \partial u_\alpha / \partial x^i + \Lambda^\beta_{\alpha i} u_\beta$ where the $\Lambda^\beta_{\alpha i}$ are determinate functions of x^h.*"
> (Cartan 1938,151)

The existence of elementary particles irrevocably seems to be attached to the existence of spinors. The existence of elementary particles hence seems to be bound to the possibility to consider the respective space to be flat. We have to envisage a *conditional existence* of fundamental particles [57]. They might exist as far as the respective location may be considered flat.

9.3.2. The expansion of the Universe: another example of conditional existence

Comparison with a similar problem occurring with the *expansion of the Universe* could provide a suggestion how to resolve this conundrum. The expansion while obviously concerning the Universe as a whole according to the reality of astronomical observations does not apply to our solar system, nor does it apply to the Milky Way nor to clusters of galaxies. Why? The expansion of space is a conditional assertion. It has been derived from the Friedmann-Walker metric *under the condition* that space be homogeneous. The above examples but are by no means representative for a homogeneous space. The phenomenon of expansion applies to the Universe as a whole only if statistics on this level makes space to appear homogeneous.

This does not mean that reality is guided by mathematical assumptions. It only says that what we observe depends on the frame that is conditioning observation.

We have to question the conception of unconditional *permanence* that lurks behind the european philosophical conception of *substance*. With the modern picture of transmutation of elementary particles into one another the conception of permanence has been severely weakend. But even before the appearance of elementary particles Eddington in his 1923 book on several occasions did remind how deeply the conception of permanence did influence the mathematical conception of classical mechanics.

9.3.3. Fermionic fundamental particles: another example of conditional existence

The conception of an *unconditional existence* of elementary particles would constitute an absolute object analogous to the Newtonian filiation of time as divorced from space. We conjecture that the *existence* of fermionic fundamental particles is bound to the condition that space may be considered to be flat. [58]

For most considerations involving fundamental particles in General Relativity this condition is of no major relevance. J. Ehlers in his 2007 account on General Relativity underlines the existence of a Lorentzian metric to be the most basic assumption of General Relativity. It implies the approximate validity of SRT in spacetime regions which are small compared to the time and distance scale set by the curvature of spacetime. ”*Even in neutron stars this scale is much larger than the scales relevant for the properties of bulk matter, atoms or nuclei. Therefore equations of state, cross sections, transport coefficients etc . derived from quantum theory can be incorporated into the classical matter models used in General Relativity in spite of the fact that these theories are in principle incompatible.”(Ehlers 2007,92)*

We therefore conjecture that fermionic matter finds its location within the antisymmetric contraction of the Riemann-Christoffel tensor provoking the condition $ds^2 = 0$.

9.4. Local position invariance (LPI)

There is some interesting corollary that underpins the view argued here [59]:

- The view argued here in a natural way implies *local position invariance (LPI)*, the 3rd ingredient within Einstein’s equivalence principle EEP. LPI states: ”The outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed.” (Will 2014, Introduction). Since General Relativity according to Eddington is describing observation as a process of *identification* of the classical contravariant measuring entities with the covariant variables of the space concept the LPI necessarily is fullfilled.

Attempts (e.g. L.Infeld and B.L.van der Waerden) to avoid this impossibility by decoupling the spinor transformation properties from their geometrical origin were rejected by Cartan as ”geometrically and even physically so startling” (french: ”choquant”)(Cartan 1938,151)

[57] Let us remind that we know this construct from the *expansion* of the universe that is bound to the condition that this space is homogeneous. The expansion thus applies to the universe as a whole which appears to be homogeneous but it does not apply to our solar system or to the galaxy or to even clusters of galaxies (see sect.9.3.2, p.42).

[58] The conditional existence of fundamental particles is compatible with the conclusions of Gibbons et.al.(1977): *”The derivation of these results involves abandoning the idea that particles should be defined in an observer independent manner.”*

[59] implying that General Relativity and the respective observations trace the physiognomy of Riemannian space when coupled with the condition of the possibility to measure

- Will 2014 in his compilation of the various tests scrutinizing General Relativity presented a position of Einstein: "He famously stated that if the measurements of light deflection disagreed with the theory he would *"feel sorry for the dear Lord, for the theory is correct!""* (Will 2014, Introduction). We underline that the conjecture argued here gives full appreciation to Einstein's view. General Relativity is not articulating a law of the Lord but in Eddington's view is unfolding a theory on measurement. This in some way is reflected in Will's additional comment:"But compared to the inner consistency and elegance of the theory, he regarded such empirical questions as almost secondary." It is the consistency of the space concept that is reflected by the *inner consistency* of the theory.

- Will in the conclusion that he draws from *The confrontation between General Relativity and Experiment* (Will 2014,conclusions) states: *"General Relativity has held up under extensive experimental scrutiny. The question then arises, why bother to continue to test it?"* And he continues: *"the predictions of general relativity are fixed; the pure theory contains no adjustable constants so nothing can be changed. Thus every test of the theory is either a potentially deadly test or a possible probe for new physics."*

- Will's conclusions state that *"all attempts to quantize gravity and to unify it with the other forces suggest that the standard general relativity of Einstein may not be the last word."* This in our opinion might be a fallacy. General Relativity and the theory of elementary particles by assumption are handling distinct realms of physics: General Relativity is putting its emphasis on the determination of a *geodesics* which is calculated from an invariant length element generally assumed to be nonzero [60]. This has been anchored by imposing a respective a priori symmetry on the Riemann-Christoffel tensor which lets the metric $g_{\mu\nu}$ appear to be *symmetric* (including light rays as a boundary phenomenon). The theory of elementary particles as compiled in the Standard Model and as seen thru the glasses of Cartan's concept of complex flat space on the contrary is based on *antisymmetric* objects that imply $ds^2 = 0$ and that do not admit any trajectory. Only such isotropic objects imply the existence of spinors which the elementary fermions are identified with. The validity of Einsteins General Relativity and that of the standard theory of elementary particles thus genuinely have nothing to do with one another [61]. The former one is describing the *symmetric* aspects of matter, the other one the *antisymmetric* aspects of matter. Both theories operate on genuinely excluding realms described by either $ds^2 \neq 0$ or $ds^2 = 0$. Both may prove to be correct under its respective assumptions without putting into question the other one.

10. Mass and the symmetric and antisymmetric nature of matter

We conjecture matter to consist of a *symmetric* sector and an *antisymmetric* sector.

The symmetric sector is described by the gravitational interaction of General Relativity, complemented by the alternative description delivered by Quantum Mechanics. Both these forms of symmetric matter genuinely are equipped with mass.

The *antisymmetric* sector is responsible for the phenomenon of elementary particles. These objects genuinely are massless as is supposed in the Standard Model before the advent of the Higgs field.

Mass hence seems to be a phenomenon related to the symmetric part of matter. The question of why fermions in the reality of experiments acquire mass then reduces to the question where the description of antisymmetric objects overlaps with features assigned to the symmetric part. This points to the fundamental polar.

We suggest that the mass of fermions be generated by the fundamental polar, $\xi^T C\xi$, for several reasons:

Stressing the analogy with QED, $\xi^T C\xi$ corresponds to a Feynman diagram with the boson line truncated. This is but the term representing mass in mass renormalization (Schweber 1962,TBD).

Expressed in covariant variables this term corresponds to the combination of a left-handed and right-handed contribution, just the way mass is expressed in the Standard Model (Shifflett,2015)

Expressed in contravariant variables this term represents the sum of squares known from SRT to be the ingredients of mass.

The fundamental polar according to this view would provide the scalar field corresponding to the Higgs field.

This supposed to be realistic the fascinating perspective would emerge that mass would be the genuine invariant from which the interactions $\xi^T C \underset{(p)}{X} \xi$ derive as its irreducible representations.

[60] with the exception of photons.

[61] Some overlap exists in the trajectory of light which in the view of General Relativity represents a boundary phenomenon and which from the view of elementary particles stems from a bosonic $U(1)$ field characterizing an interaction. Only in the ray approximation of classical electrodynamics may this field be represented by a trajectory.

The question remaining to be answered would then be how mass would get distributed to the individual spinor components.

11. Conclusions

The fondament of physics contrary to common belief is not the comparison of the results of its theories against an external entity dubbed Nature. Fact is: For every realm of physics physicists have postulated its own space concept. It serves two purposes: (i) it allows to construct an invariant that serves as a measuring stick (ii) it allows to encode the condition of the possibility to measure. It is remarkable to observe how the seesaw of experimental design and theoretical expectation in all these realms in a straightforward manner did converge towards tracing the physiognomy of the underlying space concept as minted by the condition of the possibility to measure. Basic for this success is what we call the *inversion of measurement*: instead of comparing with some external entity dubbed Nature the experimentally found objects get identified with the mathematical objects emerging from the condition of the possibility to measure: (i) the gravito-inertial field gets identified with the metric in Riemannian space (ii) the experimentally found fundamental fermions get identified with the components of a spinor in various complex dimensions (iii) the quantum mechanical objects have become identified with the mathematical support of the generators of translations and Galilei-transformations (iv) Maxwell's electromagnetic field tensor gets identified with the bivector of flat space. Physicists experimentally find the objects that are predetermined by the condition of the possibility to measure. In case of General Relativity they observe the BH's, in case of elementary fermions they find the leptons and quarks to be identified with spinor components etc. Matter and space are conditioning one another.

The *inversion of measurement* is getting rid of the metaphysical claim of an entity dubbed Nature which is supposed to exist independent of physicists and which physicists claim to measure with their apparatuses. It allows to determine the pamorama of physics to be built on distinct space concepts each delivering its own invariant needed to provide an equivalent for the measuring stick of classical physics. It allows to determine the relation of Quantum Mechanics and General Relativity as well as the location of elementary particle physics in the edifice of General Relativity. It allows to find out that QED is not a quantum theory determined by Plancks action quantum $\hbar$ despite the fact that the Feynman rules are interspersed with occurrences of $\hbar$.

The predefinition of the theory within the frame of the space concept led Eddington, the *most reknowned physicist of his time* (Chandrasekhar 1983,39), whose 1923 thorough analysis of Einstein's General Relativity till the 1970's reached eleven editions, to call General Relativity a *put-up job*:

> ' 'The whole thing is a vicious circle. The law of gravitation is - a put-up job." (Edd 1928,145)

Physicists according to Eddington get out nothing they did not put in beforehand. We show that Eddington's insight derived from General Relativity applies to the other realms of pysics as well. Measuring men encounter only themselves and their obsession to approach the world by measuring. This insigt is repeating what Kant published in 1783 in his Prolegomena:

> "Der Verstand schöpft seine Gesetze (a priori) nicht aus der Natur, sondern schreibt sie dieser vor."(Kant 1783,91)

Kant called this a second Copernican revolution. What Kant deduced as a result of a philosophical investigation Eddington derived by analysing the mathematical structure of General Relativity. Our analysis of the realms of physics besides General Relativity - elementary particle physics, electrodynamics and Quantum Mechanics - is strengthening this view.

To get rid of the metaphysical foundation of physics by no more referring to Nature is no loss. The journey into the physical exploitation of different space concepts enabled physicists to lay the foundation for ever richer technologies.

A. The standard model of elementary particle physics and complex flat space as described by reflections

A.1. Going complex: spinors and Cartan's representation of flat space

Whereas Dirac in 1928 introduced spinors into physics by trial and error when guessing the mathematical form the Schrödinger equation of Quantum Mechanics had to take on to fullfill the requirements of a relativistic treatment Cartan gave a consistent mathematical treatment starting in 1913 with the systematic investigation of group properties and culminating 1938 in his groundbreaking book *Leçons sur la théorie des spineurs* (Cartan 1938), translated to English in 1966 *The Theory of Spinors* (Cartan 1981).

Spinors according to Cartan are *geometrical* entities. They appear when a real space which is characterized by a pure quadratic form is considered to be complex. At this moment vectors of length zero are entering the stage. In real flat space objects with length zero are nul objects with all their components necessarily being zero. [62]

Making the transition from real to complex the unit basic vectors of real flat space automatically will become objects with length zero, called *isotropic* vectors. When spanning complex flat space by isotropic vectors *spinors* appear. The spinor components are the coefficients needed to secure that by adding new dimensions the resulting vectors stay isotropic.

The successive endorsement of new dimensions leads to a recursive formula which makes the spinor components significantly deviating from the behaviour of vector components. Whereas vector components have a single index referring to the space dimension the spinor components own a *compound index* which refers to either a subset of or all of the dimensions involved. For one complex dimension corresponding to real dimension $n = 2$ or 3 the spinor components are (ξ_0, ξ_1). For two complex dimensions $(n = 4, 5)$ there are four components $(\xi_0, \xi_1, \xi_2, \xi_{12})$, for three complex dimensions we find 8 components $(\xi_0, \xi_1, \xi_2, \xi_3, \xi_{12}, \xi_{13}, \xi_{23}, \xi_{123})$ and so forth. For complex dimension ν corresponding to real dimension $n = 2\nu, 2\nu + 1$ the number of spinor components grows with 2^ν. [63]

A.2. Mathematical basics

A.2.1. The transition from real to complex linear space

Real n-dimensional flat spaces E_n are constituted by the existence of a quadratic form [64] $\Phi = (x_r^1)^2 + \ldots + (x_r^n)^2$. The quadratic form characteristic for a *complex* space is taken by Cartan to be

$$F \equiv z_i z_{i'} + z_0^2 \qquad (i, i' = 1 \ldots \nu) \tag{117}$$

where $z_i, z_{i'}$ are vectors representing paired complex dimensions. z_0 designates an *unpaired* dimension taken to be real by assumption.

Choosing z_i and $z_{i'}$ to be the complex conjugate to each other one easily effects the transition to a real space of dimension $2\nu + 1$. Setting the unpaired coordinate z_0 identically to zero leads to the real space of dimension 2ν. The spinor components are not affected by this transition. The same spinor applies to spaces with and without an unpaired dimension.

A.2.2. The emergence of isotropic *vectors*

Switching to complex spaces introduces new geometrical objects that are not known from real euclidean spaces: objects of measure zero, in our case vectors of length zero, without all components being zero. Such *isotropic* objects provide the geometrical foundation of spinors.

The transition from complex spaces with coordinates x^0, x^1, x^2, $\ldots x^\nu$, $x^{1'}$, $x^{2'}$, $\ldots x^{\nu'}$ to *real* euclidean spaces with an orthogonal coordinate system x_r^0, x_r^1, x_r^2, $\ldots x_r^n$ is done by choosing the basis vectors $\vec{e}^{\,i}, \vec{e}^{\,i'}$ as complex conjugate and

[62] Another situation is produced if the space is endowed with a pseudo-euclidean metric like in SRT. This allows for objects like the photon wave vector k_μ with the property $k_\mu k^\mu = k_0^2 - \vec{k}^2 = 0$.

[63] In the Minkowski space of SRT a special situation arose: the four components of the Dirac spinors ressemble the four components of the vectors in this space. This made it easy to consider the distinction between spinors and vectors to reflect a purely technical difference in transformation properties which unhappily can't be avoided.

[64] The subscript *r* denotes coordinates in real space.

establishing a pairwise mapping between the correlated complex basis-vectors $\vec{e}^{\,i}, \vec{e}^{\,i'}$ and the real basis vectors $\vec{e_r}^{\,2i-1}, \vec{e_r}^{\,2i}$:

$$\vec{e}^{\,i} \quad \leftrightarrow \quad \frac{1}{2}(\vec{e_r}^{\,2i-1} + i\,\vec{e_r}^{\,2i}) \tag{118}$$

$$\vec{e}^{\,i'} \quad \leftrightarrow \quad \frac{1}{2}(\vec{e_r}^{\,2i-1} - i\,\vec{e_r}^{\,2i}) \tag{119}$$

Taking the real basis vectors as orthonormal, $(\vec{e_r}^{\,i}\vec{e_r}^{\,k}) = \delta_{ik}$, the basis vectors of the complex space automatically get *isotropic*

$$(\vec{e}^{\,i})^2 = (\vec{e}^{\,i'})^2 = 0 \tag{120}$$

and orthogonal

$$(\vec{e}^{\,i}\vec{e}^{\,k}) = (\vec{e}^{\,i'}\vec{e}^{\,k'}) = (\vec{e}^{\,i}\vec{e}^{\,k'}) = 0 \qquad \text{for } (i \neq k) \tag{121}$$

The one crucial exception are the basis vectors that form a complex conjugate pair. They are *not orthogonal* but fulfil

$$\vec{e}^{\,i}\vec{e}^{\,i'} = 1/2 \tag{122}$$

The unpaired *real* basis vector is orthogonal to all other basis vectors. It is normalized to 1:

$$(\vec{e}^{\,0})^2 = 1 \tag{123}$$

The appearance of *isotropic* vectors, i.e. vectors of length zero [65] according to Cartan is the paradigmatic geometrical base of the existence of spinors.

A.2.3. Spinors are the constituting coefficients of the isotropic ν-plane

The isotropic vectors span a hyperplane of maximal dimension ν, the *isotropic $\nu-$plane*. It is the same in $E_{2\nu+1}$ as in $E_{2\nu}$.

The explicit geometrical construction follows a recursive procedure. The coefficients are the components of a spinor ξ_α, where α signifies a compound index.

To understand the geometrical nature of spinors and the nature of the compound index it makes sense to take a short glance on this recursive procedure. It proceeds by a successive nesting of linear forms η_α that determine the isotropic ν-plane:

$$\eta_0 \equiv \xi_0 x^0 + \sum_k \xi_k x^k = 0 \qquad (\text{k=1} \ldots \nu) \tag{124}$$

$$\eta_i \equiv \xi_0 x^{i'} - \xi_i x^0 + \sum_k \xi_{ik} x^k = 0 \tag{125}$$

$$\eta_{ij} \equiv \xi_i x^{j'} - \xi_j x^{j'} + \sum_k \xi_{ijk} x^k = 0 \tag{126}$$

$$\eta_{ijk} \equiv \xi_{ij} x^{k'} + \xi_{jk} x^{i'} + \xi_{ki} x^{j'} - \xi_{ijk} x^0 + \sum_h \xi_{ijkh} x^h = 0 \qquad \text{etc.,} \tag{127}$$

beginning with the constants ξ_i and ξ_{ij} and supplementing additional coefficients by agglomerating the single indices to ever higher nested *compound* indices $\alpha = i_{k_1} i_{k_2} \ldots$ which reflect the nesting status:

$$\xi_0 \xi_{ijk} = \xi_i \xi_{jk} - \xi_j \xi_{ik} + \xi_k \xi_{ij} \tag{128}$$

$$\xi_0 \xi_{ijkh} = \xi_{ij} \xi_{kh} + \xi_{jk} \xi_{ih} + \xi_{ki} \xi_{jh} \qquad \text{etc.} \tag{129}$$

There are 2^ν coefficients ξ_α that provide the components of the spinor ξ.

The index α of a spinor ξ_α is a *compound* of single indices that are related to the coordinate axes [66]. Thus besides the single component ξ_0 and the single indexed components ξ_i $(i = 1, 2, \ldots \nu)$, that formally ressemble the component structure familiar from vectors, spinors in general have additional components $\xi_{i_1 i_2 \ldots i_p}$ $(p = 2, \ldots, \nu)$. They have the property of either changing sign or being unaltered under odd or even *permutations* of the indices.

These spinor components provide the mathematical base for what physically will be called fermions.

[65] Physics is a measuring discipline and the scalar product is a basic tool to describe its objects. Introducing objects with no measuring protocol at hand is what made spinors a strange experience for physicists.

[66] For example a spinor for $\nu = 2$ (E_4 and E_5) is composed by $2^\nu = 4$ components (ξ_0, ξ_1, ξ_2, ξ_{12}); a spinor for $\nu = 3$ (E_6, E_7) has $2^\nu = 8$ components (ξ_0, ξ_1, ξ_2, ξ_3, ξ_{12}, ξ_{13}, ξ_{23}, ξ_{123}).

A.3. The emergence of the Dirac equation

A.3.1. The defining equation of spinors

Besides the spinors we have 2^ν antisymmetric linear forms $\eta_{i_1 i_2 \dots i_p}$ that when equated to zero constitute the isotropic ν-plane. Every equation of the 2^ν linear forms $\eta_{i_1 i_2 \dots i_p}$ does establish a relation between the spinor components and the coordinates x^0, x^i, $x^{i'}$, which is linear in the coordinates and linear in the spinor components. Eq.(124) till (129) addressed them as a matrix consisting of spinor components operating on the vectors $(x^0, x^i, x^{i'})$ spanning the space $E_{2\nu+1}$. By taking the $\xi_a lpha$ to be independent variables with the spinor components ξ_α given a definite order and reordering the set of equations along these ordered spinor components we may *invert* the set of equations:

$$X\xi = 0 \tag{130}$$

This is the *defining equation* for the *isotropic ν-plane*. It describes the spinor components ξ_α that provide the coefficients that allow to constitute this plane. X is a matrix whose elements, except in cases where they are zero, are - apart perhaps from the sign - equal to one of the coordinates $x^1, \dots, x^\nu, x^{1'}, \dots x^{\nu'}$, which may be regarded as the contravariant components of a vector $\boldsymbol{x}$. [67]

By equation (130) each vector $\boldsymbol{x}$ gets *associated* with a matrix X of rank 2^ν with $2^{2\nu}$ elements.

The switch to convert vectors into matrix representations [68] is an essential backbone of spinor theory. In spinor space the role of the *vectors* is taken over by *associated matrices* [69]. For $\nu = 2$ and real basis vectors we recover the familiar Dirac representation of a vector p^μ by $\not p = \gamma_\mu p^\mu$.

There are some important rules that regulate the relation between vectors and their associated matrices [70]. We especially need one: let X and Y be the associated matrices of the vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ then the scalar product of both vectors gets associated with a matrix

$$(\boldsymbol{x}\boldsymbol{y}) = \frac{1}{2}(XY + YX) \tag{132}$$

If two vectors are orthogonal their associated matrices hence will anticommute. *Anticommuting* matrices in the spinor calculus occupy the role of *orthogonal* basis vectors.

The square of a matrix X associated to the vector $\boldsymbol{x}$ thus equals the square of the vector: $X^2 = \boldsymbol{x}^2$. This allows to infer that the vectors that satisfy eq.(130) are isotropic because

$$XX\xi = X^2\xi = \boldsymbol{x}^2\xi = 0 \qquad \text{which implies } \boldsymbol{x}^2 = 0 \tag{133}$$

Two important rules are to be noted: let X and Y be the associated matrices of the vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ then the scalar product of both vectors gets associated with a matrix

$$(\boldsymbol{x}\boldsymbol{y}) = \frac{1}{2}(XY + YX) \tag{134}$$

If two vectors are orthogonal their associated matrices hence will anticommute. *Anticommuting* matrices in the spinor calculus occupy the role of *orthogonal* basis vectors. As an immediate consequence we get te relation:

$$(x^2)\xi = XX\xi \tag{135}$$

stating that the vectors of the isotropic ν-plane $(X\xi = 0)$ necessarily are isotropic $(x^2 = 0)$.

---

[67] Taking $\nu = 2$ and arranging the components of the spinor according to $\xi_0, \xi_1, \xi_2, \xi_{12}$ we get (Cartan 1981, 81):

$$X = \begin{pmatrix} x^0 & x^1 & x^2 & 0 \\ x^{1'} & -x^0 & 0 & x^2 \\ x^{2'} & 0 & -x^0 & -x^1 \\ 0 & x^{2'} & -x^{1'} & x^0 \end{pmatrix} \tag{131}$$

[68] Note that we will eventually continue to use the name vector when we are speaking of the associated matrix of the vector.

[69] We follow Cartan by using a capital letter X to indicate the associated matrix of a vector x.

[70] E.g. in three dimensions the bivector constructed from the vectors $\vec{x}$ und $\vec{y}$ with components $x_2y_3 - x_3y_2$, $x_3y_1 - x_1y_3$, $x_1y_2 - x_2y_1$ is associated with a matrix $\frac{i}{2}(XY - YX)$. The determinant of an associated matrix X may be equated with the negative scalar product of the vector x. (Cartan 1981,44)

A.3.2. The representation of vectors by associated matrices

Let the associated matrices of the *isotropic* basis vectors $\vec{e}^{\,0}, \vec{e}^{\,i}, \vec{e}^{\,i'}$ be the matrices $H_0, H_i, H_{i'}$.
The associated matrices H_i and $H_{i'}$ of the isotropic basis vectors referring to the dimension i and i' geometrically represent a reflection on the hyperplane that is perpendicular to the respective basis vectors $\vec{e}^{\,i}, \vec{e}^{\,i'}$.
Each vector $\boldsymbol{x}$ then may be represented by the associated matrix

$$X = x^0 H_0 + x^1 H_1 + \ldots + x^\nu H_\nu + x^{1'} H_{1'} + \ldots + x^{\nu'} H_{\nu'} \tag{136}$$

decomposed in terms of reflection operators.
The decomposition (136) exhibits the central role that *reflections* are playing in representations based on spinors.
From rule (132) and from eq.(120) - (123) we get rules for the reflection matrices:

$$H_0^2 = 1,\; H_0 H_k = -H_k H_0,\; H_0 H_{k'} = -H_{k'} H_0 \qquad (k \neq 0) \tag{137}$$

$$H_i H_k = -H_k H_i,\; H_{i'} H_{k'} = -H_{k'} H_{i'} \tag{138}$$

where the last equation for $i = k$ means $H_i^2 = H_{i'}^2 = 0$ [71] . But for the *conjugate pairs* we get:

$$H_i H_{k'} + H_{k'} H_i = \delta_{ik} \tag{139}$$

A.3.3. Switching to real space: the emergence of the Clifford algebra

Switching from an isotropic to the *orthonormal* system of *real* unit basis vectors $(\vec{e}_r^{\,i} \vec{e}_r^{\,k}) = \delta_{ik}$ of Sec.(A.2.2) we get the real equivalents A_r of $H_i, H_{i'}$.
e.g for $\nu = 2$:

$$A_1 = H_1 + H_{1'} \tag{140}$$

$$A_2 = i(H_1 - H_{1'}) \tag{141}$$

$$A_3 = H_2 + H_{2'} \tag{142}$$

$$A_4 = i(H_2 - H_{2'}) \tag{143}$$

Using rule (132) for the associated matrices A_i we immediately obtain the commutation rules

$$A_i A_k = -A_k A_i \qquad (i \neq k); \qquad (A_i)^2 = 1 \tag{144}$$

These operators thus form a *Clifford* algebra (Cartan 1981,83). For $\nu = 2$ these are the well known Dirac γ-matrices [72].
The effect on the vector X of a reflection A in the hyperplane π normal to the unit vector A is given by the formula

$$X' = -A\, X\, A \tag{145}$$

The effect of this reflection on a spinor ξ is given by the formula

$$\xi' = A\, \xi \tag{146}$$

This operation unavoidably is two-valued since we may take either A or $-A$ as the unit vector normal to π.

[71] It is helpful to know that $H_0 = H_0^T$ and $H_{i'} = H_i^T$.

[72] For ($\nu = 2$ viz. $n = 4$) the associated matrices P correspond to the Dirac nomenclature $\not{p} = \gamma^\mu p_\mu$. The Dirac matrices γ^μ in this case are identical to the reflection operators $H_i, H_{i'}$. See Cartan (1981,134 (sect.157) for an explicit representation in configuration space).

A.4. The emergence of left- and right-handed spinors

A.4.1. Classes defined by reflection along the unpaired dimension

The reflection operator H_0 operating along the unpaired dimension z_0 that represents the extension from $E_{2\nu}$ to $E_{2\nu+1}$ acquires a special role: Operating on a spinor ξ_α it gives

$$H_0 \xi_\alpha = \pm \xi_\alpha \tag{147}$$

depending on whether the *compound* index α contains an *even* or an *odd* number of *single* indices.

The spinors in spaces with dimension 2ν hence decompose into two classes of *semi-spinors* whose components have either an *even* or an *odd* number of single indices in their compound index. This decomposition is the mathematical basis for the *existence of two classes* of either left-handed or right-handed fermions.

A.4.2. Semi-spinors in even-dimensional spaces: distinguishing left- and righthanded spinors

Let us pass to $E_{2\nu}$: we may order the spinor components according to first noting all components with even number of subindices, followed by all components with uneven number of subindices. Then because all the reflection operators $H_i, H_{i'}$ commute with H_0, we can distinguish two groups of semi-spinors, which by rotation get transformed onto itself, representing left-handed and right-handed spinors. For $\nu = 2$ (E_4) we get the semi-spinors of the first type (ξ_0, ξ_{12}) and of the second type (ξ_1, ξ_2), for $\nu = 3$ (E_6) we get the first type to be $(\xi_0, \xi_{12}, \xi_{13}, \xi_{23})$ and the 2nd type to be $(\xi_1, \xi_2, \xi_3, \xi_{123})$.

A.4.3. Antiparticles and right/left-handed semi-spinors

In the nomenclature of the Standard Model [73] we have the following relations between Dirac spinors Ψ and left/right handed (Weyl) semi-spinors Ψ_L, Ψ_R:

$$\Psi = \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} \tag{148}$$

The antiparticles are denoted by upperscript "c" and defined by:

$$\Psi^c = -i\gamma^2 \Psi^*; \qquad \text{leading to} \qquad \Psi^c_L = -i\sigma^2 \Psi^*_R; \qquad \Psi^c_R = i\sigma^2 \Psi^*_L \tag{149}$$

Expressed in semi-spinor components this reads:

$$\Psi^c_{L1} = -\Psi^*_{R2}\,, \qquad \Psi^c_{L2} = \Psi^*_{R1}\,, \qquad \Psi^c_{R1} = \Psi^*_{L2}\,, \qquad \Psi^c_{R2} = -\Psi^*_{L1} \tag{150}$$

The antiparticle components of left-handed particles in the Standard Model are thus the complex conjugate of the components of right-handed ones and vice versa, with their position interchanged, $1 \leftrightarrow 2$, and with each occurrence of $L1$ adopting a minus sign. [74]

A.5. Antisymmetric objects in flat space: p-vectors

The spinors are totally antisymmetric objects. We may introduce another type of a totally antisymmetric object, the *p-vector*. [75]. p-vectors are the antisymmetric extension of the vector concept. For $p = 1$ we get the familiar vector components, for $p = 0$ we take the unit 1. For $p = 2$ we will encounter e.g. the antisymmetric field tensor $F_{\mu\nu}$.

---

[73] $\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \tilde{\sigma}^\mu & 0 \end{pmatrix}$

$$\sigma^\mu = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right], \qquad \tilde{\sigma}^\mu = [\sigma^0, -\sigma^1, -\sigma^2, -\sigma^3]$$

[74] For conjugate spinors see Cartan, p.100 ($E_{2\nu+1}$), p.123 ($E_{2\nu}$)

[75] Bivectors vw correspond to the wedge product $v \wedge w$. Trivectors vwu to $v \wedge w \wedge u$ (s. Baylis 4.2.1, in Rafal Ablamowicz, Garret Sobczyk, Lecture on Clifford (Geometric) Algebras and Applications, R. Ablamowicz and G. Sobczyk, (Eds.) Birkhauser, Boston, 2004 (ISBN 0-8176-3257-3) (Baylis_44.png)

The associated matrices of these antisymmetric objects $\underset{(p)}{X}$ are the equivalent of the *bosons* that we encounter in the Standard Model. Like the *fermions*, they are structures emerging from isotropic hyperplanes, i.e. from objects with length zero. This makes these bosons to genuinely become massless objects.

Lets recapitulate a short explanation of p-vectors. Its components are composed by the products of the n contravariant (or covariant) components of p vectors $x, y, \ldots, z$ (Cartan 1981,16). By building the matrix

$$\begin{pmatrix} x^1 & x^2 & \ldots & x^n \\ y^1 & y^2 & \ldots & y^n \\ \ldots & \ldots & \ldots & \ldots \\ z^1 & z^2 & \ldots & z^n \end{pmatrix} \tag{151}$$

the components of the p-vector will consist of all the $(p \times p)$-determinants that may be built from choosing arbitrary p columns out of the n columns and putting the newly defined components into a definite order.

The *associated* p-vector $\underset{(p)}{X}$, i.e. its associated matrix, can be represented by the matrix

$$\frac{1}{p!} \sum \pm X_{i_1} X_{i_2} \ldots X_{i_p} \tag{152}$$

where X_i $(i = 1, \ldots, p)$ represent the associated matrices of the p vectors x_i and the sum extends over all permutations of the indices $1, 2, \ldots, p$ the sign being $+$ or $-$ according to whether the permutation is even or odd. Such a matrix has as elements linear combinations of the components of the p-vector. If the p vectors are orthogonal in pairs, the matrix associated with the p-vector equals $X_1 X_2 \ldots X_p$. The matrices associated with two distinct p-vectors are themselves distinct.

p-vectors are irreducible with respect to the group of rotations. The elements of $\underset{(p)}{X}$ are linear combinations of the components of the p-vector. By a rotation they are linearly transformed amongst themselves. The linear combinations which give the elements are linearly independent with respect to the ${}^{n}C_p$ components of the p-vector (Cartan 1981,85).

Under the reflection on the hyperplane normal to the unit vector a the p-vector transforms as

$$\underset{(p)}{X}' = (-1)^p A \underset{(p)}{X} A \tag{153}$$

In writing a *rotation* as the product of an even number of reflections, $S = A_{2k} A_{2k-1} \ldots A_2 A_1$ we easily get the formulae

$$\underset{(p)}{X}' = S \underset{(p)}{X} S^{-1}; \qquad \xi' = S\xi \tag{154}$$

and for a reversal we get

$$\underset{(p)}{X}' = (-1)^p T \underset{(p)}{X} T^{-1}; \qquad \xi' = T\xi \tag{155}$$

where T is the product of an uneven number $\leq 2\nu + 1$ of matrices associated with unit vectors.

B. The Standard Model: a hybrid of reflections and rotations

B.1. Analyzing the Standard Model Lagrangian

To get a deeper insight into this alternate representation let's remind the structure of the Standard Model. For orientation reasons we use the Standard Model Lagrangian as extracted by Shifflett (2015) to be our pilot Lagrangian. This Lagrangian comprises a fundament of spinorial interaction terms corresponding to what we would expect from Cartan. On top of this basement there are two layers interlaced that represent the action of $SU(2)$ and $SU(3)$ by means of their generators.

Parity violation has been built in explicitly by referring separately to left handed and right handed spinors, indexed e.g. as e_L and e_R. The chosen setup starts from the left handed doublets representing $SU(2)$-breaking by using the semi-spinors ν_L and e_L (leptons) and u_L and d_L (quarks):

$$\text{leptons} \qquad (\bar{\nu}_L, \bar{e}_L) i\bar{\sigma}^\mu D_\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \tag{156}$$

$$\text{quarks} \qquad (\bar{u}_L, \bar{d}_L) i\bar{\sigma}^\mu D_\mu \begin{pmatrix} u_L \\ d_L \end{pmatrix} \tag{157}$$

Apart from having the formal envelope corresponding to $SU(2)$ it reflects the typical product of two semi-spinors coupled to the associated matrix

$$i\bar{\sigma}^\mu D_\mu; \qquad \bar{\sigma}^\mu = (\sigma^0, -\sigma^1, -\sigma^2, -\sigma^3) \tag{158}$$

The 2-component semi-spinors ν_L, e_L, u_L and d_L contract into the σ-matrices thus effecting the coupling to the associated matrix D. This is the overall basic scheme as expected from Cartan's construction by means of reflections. The sum over the indices ($\mu = 0, \ldots, 3$) indicates that we are operating in spacetime, $\nu = 2(n = 4)$.

B.1.1. Interlacing reflections with rotations induced by $SU(2)$

On top of this basic scheme defined by reflections the generators of $SU(2)$ and $SU(3)$ are introduced by incorporating their action into the associated matrix $\sigma^\mu D_\mu$ by using the principle of generalized gauge invariance. Let's first have a look at the electroweak interaction:

$$\text{leptonic:} \qquad D_\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \left[\partial_\mu - \frac{ig_1}{2} B_\mu + \frac{ig_2}{2} \boldsymbol{W}_\mu\right] \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \tag{159}$$

$\boldsymbol{W}_\mu$, a $2x2$ matrix, is built up from the 3 generators of $SU(2)$

$$\boldsymbol{W}_\mu = W^b_\mu \sigma_b \qquad \sigma_b\,(b = 1, \ldots, 3)\ \text{(Pauli 2x2 SU(2) generators)} \tag{160}$$

contracting into the electroweak 2-component vector.

The field B_μ represents the generator of $U(1)$ acting as the identity with respect to the electroweak vector. The (V-A)-model became a key ingredient into the standard model when it was realized that the V-A-theory had to be applied not to the nucleons, but to the constituents of the nucleons, the quarks (see eq.(161)).

B.1.2. Interlacing $SU(3)$

The hybrid technique of interlaced spaces produces a hidden pattern when the quarks u_L and d_L get coupled to the strong interaction $SU(3)$

$$\text{quark:} \qquad D_\mu \begin{pmatrix} u_L \\ d_L \end{pmatrix} = \left[\partial_\mu + \frac{ig_1}{6} B_\mu + \frac{ig_2}{2} \boldsymbol{W}_\mu + ig\boldsymbol{G}_\mu\right] \begin{pmatrix} u_L \\ d_L \end{pmatrix} \tag{161}$$

The terms B_μ and $\boldsymbol{W}_\mu$ exert a similar action on the $SU(2)$-vector (u_L, d_L) like they do.ö in the leptonic case. But now there is a third layer on top of the other features: $\boldsymbol{G}_\mu$ contains the generators related to $SU(3)$ which makes it to be a 3×3 matrix:

$$\boldsymbol{G}_\mu = G^a_\mu \lambda_a \qquad \lambda_a\,(a = 1, \ldots, 8) \qquad \text{(Gell-Mann } 3 \times 3\ SU(3) \text{ generators)} \tag{162}$$

What spinors is this matrix acting on? Like all the other semi-spinors u_L and d_L are semi-spinors contracting into the σ^μ of eq.(158). But in addition u_L and d_L get loaden with an index (r, g, b) referring to the three basic entities of the fundamental representation of $SU(3)$. It is these color indices the generators λ_a of $SU(3)$ are contracting into.

The Standard Model thus is superimposing a layer of rotations on the basic layer of reflections.

B.1.3. Ad hoc handling of right handed semi-spinors

The inclusion of parity violation requires the different behaviour under $SU(2)$. This is the reason why the index "L" appears on the 2-component spinors in eq.(156), p.50. For sake of completeness we reproduce the way the Lagrangian makes right-handed spinors act as singlet terms:

$$\text{leptonic} \qquad \bar{e}_R i\bar{\sigma}^\mu D_\mu e_R + \bar{\nu}_R i\bar{\sigma}^\mu D_\mu \nu_R \tag{163}$$

$$\text{quarks} \qquad \bar{u}_R i\bar{\sigma}^\mu D_\mu u_R + \bar{d}_R i\bar{\sigma}^\mu D_\mu d_R \tag{164}$$

No W-boson terms representing $SU(2)$ appear in all these right-handed interactions. Consequently they act on singlets ν_R, e_R, u_R, d_R and not on the respective isospin doublets. The terms for the $U(1)$ generators B_μ are chosen to be different for all the singlets:

$$\text{quarks:} \qquad D_\mu u_R = \left[\partial_\mu + \frac{i2g_1}{3} B_\mu + ig\boldsymbol{G}_\mu\right] u_R \tag{165}$$

$$\text{quarks:} \qquad D_\mu d_R = \left[\partial_\mu - \frac{ig_1}{3}B_\mu + ig\boldsymbol{G}_\mu\right] d_R \tag{166}$$

For right-handed leptons the gluon term is missing:

$$\text{leptons:} \qquad D_\mu e_R = \left[\partial_\mu - ig_1 B_\mu\right] e_R \tag{167}$$

$$\text{leptons:} \qquad D_\mu \nu_R = \partial_\mu \nu_R \tag{168}$$

B.1.4. The transition to generators

The fact that the right-handed semi-spinors required another interaction pattern than the left-handed semi-spinors forced to handle all these interactions on a semi-spinor base. This meant to use σ- instead of γ-matrices, both still representing reflection operators. But the Cartan reflection operators σ_i are identical to the generators of $SU(2)$.

An infinitesimal $SU(2)$-transformation around the identity $S = E - idx^i\sigma_i$ is featuring the generator σ_i embedded into an associated matrix $dx^i\sigma_i$ (Tung 2003,127). With respect to $SU(2)$ we thus find an equivalence between a representation using reflection operators and rotation generators.

The shift in interpretation consists in taking the σ_i to be generators of $SU(2)$ [76] instead of reflection operators built up from Cartan's $H_i, H_{i'}$.

The symmetry $SU(2)$ entered the scene, inducing the shift from a representation of bosons by associated matrices built up by reflections to a representation by vector fields contracting into generator matrices instead of associated matrices.

But the formal equivalence of a representation in terms of associated matrices or a representation by generators of rotations ends for $SU(3)$ and higher symmetries. The generators of $SU(n)$ are $n^2 - 1$ matrices with dimension n. The elementary objects are thus to be represented by vectors of dimension n which the generators contract into. The dimension of spinors according to the geometrical definition of Cartan but grows with 2^ν. There is no room for a spinor of dimension 3. What for a dimension two appears to be an isomorphism between the vector representation of $SU(2)$ and a spinor representation for $\nu = 1$ or a semi-spinor representation for $\nu = 2$ fades away for higher dimension. [77] There is no straightforward correlation between the Cartan dimension ν and the symmetries $SU(\nu)$.

Generalized gauge invariance then served as a guide to collect $U(1)$, $SU(2)$ and $SU(3)$ under the same roof representing extensions of the translation generator ∂_μ [78]. These rotations determine the apperception of the Standard Model as representing $SU(3)_c \otimes SU(2) \otimes U(1)_Y$. The representation in terms of generators captured the interpretation of the Standard Model.

B.1.5. The gap in interpretation

Although expected to be mostly equivalent because a rotation is equivalent to two reflections a gap exists in interpretation between the Standard Model which is based mostly on rotations and the SMC based on flat space defined by reflections.

Leaving reflections and spinors confined to the basement and installing on top a layer in terms of generators of rotations contracting into vectors induced a far-reaching shift in interpretation of the Standard Model. Representing fermions as being components of vectors that contract into generators of rotations denies the emergence of particles from the space concept. They become particles that exist *in* space supporting the Newtonian view of matter.

[76] Rotations in 3 dimensions may be written as $= e^{i\frac{\theta}{2}\vec{n}\vec{\sigma}}$ where $\vec{n}$ denotes the rotation axis. This representation by reflection operators is the unique way to make the law of group multiplication of rotations transparent:

$$S_3 S_2 S_1 = e^{i\frac{\theta_3}{2}\vec{n_3}\vec{\sigma}} e^{i\frac{\theta_2}{2}\vec{n_2}\vec{\sigma}} e^{i\frac{\theta_3}{2}\vec{n_1}\vec{\sigma}} \tag{169}$$

where $\vec{n_1}, \vec{n_2}\vec{n_3}$ are the rotation axes of three subsequent rotations.

[77] In these regions there exist socalled accidential isomorphisms only between the classical Lie groups: $Spin(3) \sim SU(2), Spin(4) \sim SU(2) \otimes SU(2), Spin(6) \sim SU(4)$ (Zee 2016,563: $SO(6) \sim SU(4)$) where Spin(n) is shorthand for Spin($\boldsymbol{R}^n$). (see https://en.wikipedia.org/wiki/Spin_group).

[78] This led to the dominance of $(p = 1)$ vector- instead of multivector representations of bosons in the Standard Model.

C. QED is not a quantum theory

C.1. 2nd quantization as a back salto from Quantum Mechanics into flat space

C.1.1. The disappearance of the quantum mechanical commutator $[q, p] = i\hbar$ in QED

The commutator $[q, p] = i\hbar$ which has become so famous for quantum mechanics has been introduced into QED by quantizing the e.m. field according to (183). This directly led to the factor $\hbar$ in the propagator (184). But miraculously all the factors $\hbar$ got absorbed into the e.m. coupling constant α. This obviously is a precondition for the identification of QED as a theory resulting from an analysis of flat space which genuinely doesn't even know about $\hbar$. But it is instructive to look at the harmonic oscillator to get an impression of how this works. By setting

$$a^{\dagger} = \frac{1}{\sqrt{2}}(\frac{p}{\hbar} + iq), \qquad a = \frac{1}{\sqrt{2}}(\frac{p}{\hbar} - iq) \tag{170}$$

we easily from $[q, p] = i\hbar$ get the relation:

$$[a, a^{\dagger}]_{-} = 1 \tag{171}$$

which is independent on $\hbar$. It shows the signature of creation and annihilation operators of bosons.

The definition in eq.(111) shows that by introducing the combination $p/\hbar$ the original meaning of the covariant form k has been restored as being a *one-form* representing the generator of translations $k = -i\partial q$. The creation and annihilation operators are nothing but representations of the genuine relation that connects *covariant* and *contravariant* entities, $[\partial q, q]_{-} = 1$. This elementary commutator does express the basic multiplication rule of differential calculus. Identifying the covariant one-form k as the generator of translations $k = -i\partial q$ we get $-i[k, q] = 1$ as the elementary commutator relating tangent space and its dual. By using its fundamental postulate $p = \hbar k$ Quantum Mechanics gets the commutator to be $[q, p] = i\hbar$ [79] Thus $\hbar$ is organizing the transition from covariant to contravariant entities and vice versa, i.e. from waves to particles and vice versa. QED but does not need to invoke this transition except for convention. It is handling the distinction between wave and particles by means of commutators with different sign between fermionic and bosonic creation and annihilation operators. No $\hbar$ is needed.

C.2. The framework of QED

C.2.1. The basic assumptions of QED

We recapitulate the basic assumptions of QED:

We start with a time displacement operator $U(t, t_0)$ defined by $|t >= U(t, t_0)|t_0 >$ where $|t >$ is the state of the system at time t . $U(t, t_0)$ has to be unitary to fulfill the conditions on a probability interpretation. An infitesimal time displacement then reads

$$U(t_0 + \delta t, t_0) = 1 - \frac{i}{\hbar} H\delta t \tag{172}$$

where the Hamiltonian H has to be hermitian to guarantee U to be unitary. The parameter $\hbar$ in this expression is a pure convention to make the time displacement operator H an energy. We easily recognize the character of this convention: it serves to allow the coefficient of t to be taken as contravariant instead of a covariant entity.

The time development in the Heisenberg picture then gets controlled by (Schweber 1962,334):

$$U(t, t_0) = P\left(e^{-\frac{i}{\hbar}\int_{t_0}^{t} H_{int}(t')dt'}\right) \tag{173}$$

H_{int} is the interaction Hamiltonian in the Dirac picture and P means a specific ordering of the operators. The ordering operator P had to be introduced because the fields are made operators defined by commutation relations.

[79] with a sign change since Quantum Mechanics has to perform the transit from a covariant to the contravariant entity p.
This transition unfolds the secret of Quantum Mechanics. Quantum Mechanics is proposing a surrogate in flat space for the gravito-inertial field of General Relativity by postulating a connection between contravariant and covariant entities $p^{\mu} = g^{\mu\nu}\hbar k_{\mu}$. By its very nature General Relativity establishes such a connection by means of the gravito-inertial field as $p^{\mu} = g^{\mu\nu}p_{\mu}$. Quantum Mechanics by operating in flat space hasn't to care about the distinction between covariant and contravariant entities.

Inserting the Hamiltonian of QED

$$H_{int} = e\tilde{\psi}\gamma^{\mu}A_{\mu}\psi \tag{174}$$

with $e\tilde{\psi}\gamma^{\mu}\psi$ the current that gets coupled to the e.m. potential A_{μ} (Schweber 1962,441), switching to *densities* and changing the time integral from dt to dx_0 we get for the S-matrix $S = U(\infty, -\infty)$:

$$S = P\left(e^{-\frac{ie}{\hbar c}\int d^4x\tilde{\psi}\gamma^{\mu}A_{\mu}\psi}\right) \tag{175}$$

We note that the Hamiltonian which in QED had to be postulated possesses the characteristic form that a Cartan invariant in spinor space automatically must take on. Moreover the kinematic variable A_{μ} takes the characteristic form $\not{A} = \gamma^{\mu}A_{\mu}$ of a Cartan associated matrix.

Our conjecture that QED is representing nothing but flat space requires that the amplitudes do not depend on $\hbar$. To show that $\hbar$ in fact is completely absorbed into the e.m. coupling constant $e^2/\hbar c$ we have to investigate the appearance of propagators within the feynman diagrams (Schweber 1962,471)

C.2.2. Quantization of fermion fields

Developping the Schrödinger field operators $\psi, \tilde{\psi}$ in terms of a complete set of eigenfunctions $w_n(x)$ of the free-particle Dirac equation (Schweber 1962,219) introduces the spinors $w^r(p)$ $(r = 1, 2, 3, 4)$

$$\psi(x) = \frac{1}{\sqrt{V}}\sum_n \sqrt{\frac{m}{|E_n|}}b_n w_n(x) \tag{176}$$

$$w_r(x) = w^r(p)e^{ipx}; \qquad (r = 1, 2) \tag{177}$$

$$w_r(x) = w^r(p)e^{ipx}; \qquad (r = 3, 4) \tag{178}$$

Quantization is achieved by interpreting the expansion coefficients as creation and annihilation operators in order to fulfill the Pauli exclusion principle:

$$[b_n, b_m^*] = \delta_{nm}; \qquad [b_n, b_m] = [b_n^*, b_m^*] = 0 \tag{179}$$

In passing we note that in QED the Pauli principle had to be postulated ad hoc. But (179) are the commutation relations of *reflection operators* $H_i, H_{i'}$ which *automatically* induce the Pauli principle (see sect (3.2.2)), p.14. The fields then obey the equal time commutation relation

$$[\psi(x), \tilde{\psi}(x')]_{+\,x_0=x_0'} = \gamma_0\delta^{(3)}(x - x') \qquad \text{(all other commutators zero)} \tag{180}$$

Switching to Heisenberg operators, requiring relativistic invariance and generalizing from equal time to arbitrary space-like separations we finally get (Schweber 1962,276):

$$[\psi(x), \tilde{\psi}(x')]_{+} = -iS(x - x') \qquad \text{for } (x - x')^2 < 0 \tag{181}$$

C.2.3. Quantization of boson fields

In QED the canonical quantization procedure for boson fields in strict analogy to Quantum Mechanics is *postulating*

$$[\Pi^{\mu}(x), A_{\nu}(x')]_{x_0=x_0'} = +i\hbar c\delta^{\mu}_{\nu}\delta(\boldsymbol{x} - \boldsymbol{x'}) \tag{182}$$

Covariantly formulated this becomes

$$[A_{\mu}(x), A_{\nu}(x')] = -i\hbar c g_{\mu\nu}D(x - x') \tag{183}$$

with $D(x - x') = \Delta(x - x')$ for boson mass $\mu = 0$ (Schweber 1962,276).

C.2.4. *Decomposition into normal products*

The right hand sides of the commutators (181) and (183) are the *propagators*. In the series expansion of the exponential care has to be given to the ordering of the operators. Decomposing the S-matrix into *normal products* $N(,)$ in which all the creation operators standing to the left of all annihilation operators guarantees that for any given initial and final state with a definite number of free particles with specified spins and momenta there will be one and only one normal product with a nonzero matrix element between these states. The enormous usefulness of Feynman diagrams results from the fact that they are a concise way of representing a normal product (Schweber 1962,435). The necessary transit from the Dyson time ordered product P in (173) to normal products gives rise to the appearance of the right hand sides of (181) $-1/2\, S_F(x-x')$ and (183) $-1/2\, \hbar c\, D_F(x-x')g_{\mu\nu}$ as *propagators* in the *Feynman rules* [80] in configuration space (Schweber 1962,471). In *momentum space* they read (Schweber 1962,478):

$$\text{boson propagator:} \qquad -\frac{i\hbar c}{(2\pi)^4}\,\frac{1}{k^2+i\epsilon}\,g_{\mu\nu} \tag{184}$$

$$\text{fermion propagator:} \qquad \frac{i}{(2\pi)^4}\,\frac{1}{\gamma\cdot p - M + i\epsilon} \tag{185}$$

The result is that even though the *time dependend* Dirac equation shows an $\hbar$ dependence, the fermionic propagator does not show up any relation to $\hbar$. The photon propagator but shows a factor $\hbar c$. This amounts to a factor $\sqrt{\hbar c}$ per knot for any *internal* photon line, since such a line does connect to two knots.

C.2.5. *External photon lines*

There is a second source for the appearance of $\hbar$ in the amplitudes resulting from an *external* photon (Schweber 1962,478):

$$\epsilon_\mu^{(\lambda)}(\boldsymbol{k})\,\frac{(\hbar c)^{1/2}}{(2\pi)^{3/2}}\,\frac{1}{\sqrt{2|k_0|}} \tag{186}$$

Every external photon accordinly contributes with a factor $\sqrt{\hbar c}$ per knot.
Thus we get the overall result, that *every* photon line, whether internal or external does contribute a factor $\sqrt{\hbar c}$ per knot.

C.3. The Feynman rules show: the squared amplitudes are independent on $\hbar$

The series expansion of the S-matrix (175) in powers of the exponent gives a factor $(\frac{-ie}{\hbar c})^n$, where n is the number of knots of the respective Feynman diagrams. The Feynman rules but inform us that there is an additional factor $\sqrt{\hbar c}$ at every knot coming either from an external or an internal photon line. *Any* knot thus brings in a factor

$$(\frac{-ie\sqrt{\hbar c}}{\hbar c})^n = (\frac{-ie}{\sqrt{\hbar c}})^n \tag{187}$$

The *squared* amplitude of the n-th order approximation thus turns out to obey a pure dependence on the e.m. coupling contant:

$$|M|^2 \sim \left(\frac{e^2}{\hbar c}\right)^n \sim \alpha^n \tag{188}$$

This is the well known base of the successfull series expansion of the scattering amplitudes of QED in powers of the e.m. coupling constant. What we would like to stress is that the appearance of $\hbar$ in the Feynman rules is spurious and gets absorbed into the e.m. coupling constant α_{em}. The following quotation might give a glance on the widespread misjudgement of the influence of Quantum Mechanics on QED:

> ”Feynman rules are the main tool of the contemporary particle theorist. These rules incorporate the basic concepts of quantum mechanics...” (Veltman 2003,246)

α_{em} but is a constant representing flat space, which does not depend on quantum mechanics. QED though evolving from Quantum Mechanics by building on associated matrices and Cartan invariants made a back salto and is based on the concept of flat space as constituted by reflections.

[80] The index ”F” is defined as $D_F(x) = +2iD^{(+)}(x)$ for $x_0 > 0$ and $D_F(x) = -2iD^{(-)}(x)$ for $x_0 < 0$ wit $(+)$ and $(-)$ the usual positive and negative frequency parts (Schweber 1962,442).

C.3.1. The spectrum of the hydrogen atom

As we indicated (sect.A.3) , the time-independent Dirac equation

$$(\not{p} - e/c \not{A} - mc)\psi = 0 \tag{189}$$

has its origin in the defining equation of spinors. As a consequence the *dash* appears which denotes that the Cartan associated matrices have to be used in place of the familiar kinematical variables.

The energy levels of the hydrogen atom without radiative corrections included may be calculated directly from the Dirac equation (189) with the result [81] (Schweber 1962,104):

$$E_{n,j} = mc^2 \left(1 + \frac{\alpha^2 Z^2}{(n' + \sqrt{(j+1/2)^2 - \alpha^2 Z^2})^2}\right)^{\frac{1}{2}} \qquad n' = 0, 1, 2 \ldots ; \, j = \frac{1}{2}, \frac{3}{2}, \ldots \tag{190}$$

No $\hbar$ dependence is found in these energy levels. The resulting formula may be approximated in two steps which are easy to follow to get the characteristic form that defines the Rydberg constant:

$$E_{n,j} = mc^2 \left(1 + \frac{\alpha^2 Z^2}{(n' + (j+1/2))^2}\right)^{\frac{1}{2}} \tag{191}$$

$$E_{n,j} = mc^2 \left(1 - 1/2 \frac{\alpha^2 Z^2}{(n' + (j+1/2))^2}\right) \tag{192}$$

Thus the emission spectrum is given by

$$\Delta E = -\frac{mc^2 \alpha^2 Z^2}{2} \left(\frac{1}{(n' + (j+1/2))^2} - \frac{1}{(n'' + (j+1/2))^2}\right) \tag{193}$$

This shows that also the results gained with the help of the Dirac equation are expressed by α and not by $\hbar$.

The factor in front of the bracket in (193) is the Rydberg constant in terms of energy. It shows no $\hbar$-dependence in spite of familiar knowledge that the Rydberg constant involves $\hbar$.

It is a special presentation of the results of QED which introduces $\hbar$ and erroneously makes QED appear to be a quantum theory. The same applies to the immediate results of the Dirac equation. We choose three examples to demonstrate how this misleading appearance of $\hbar$ comes into place, the *Rydberg constant*, the Compton scattering and the *anomalous magnetic moment* of the electron. The first case is based on the Dirac equation, the latter ones on the quantized field theory.

C.3.2. The Rydberg constant

The familiar Rydberg constant is defined to be $1/\lambda$. The calculation in the last section (sect.C.3.1) refers to no $\hbar$-dependence. But the factor in (193) conventionally gets divided by the conversion factor hc to give the familiar result

$$R_\infty = \frac{mc}{h} \frac{\alpha^2}{2} Z^2 \tag{194}$$

This result displays an *apparent dependence* on the action quantum where there is actually no dependence. The reason for the need of the action quantum is the fact that the familiar representation in terms of $1/\lambda$ insists to use a *covariant* entity which belongs to the photon wave vector k_μ. It requires the formula $p_\mu = \hbar k_\mu$ of Quantum Mechanics to translate from the contravariant energy expressed in terms of mc^2 to the covariant entity $[cm]^{-1}$.

[81] Although a consistent one-particle interpretation for the Dirac equation can be given only in the absence of interactions, the solutions of the Dirac equation in external fields play an important role in the formulation of QED.

C.3.3. Compton scattering

The amplitude of Compton scattering as given by Feynman rules in second order approximation is

$$R_{ab} = \delta^{(4)}(p_2 + k_2 - p_1 - k_1)\left(\frac{\alpha}{2\pi i}\right)\frac{1}{\sqrt{2|k_0|_2}}\frac{1}{\sqrt{2|k_0|_1}}\frac{m}{\sqrt{E(p_1)E(p_2)}}\tilde{w}^{s_2}(\boldsymbol{p}_2)\,[\text{spin-polarization terms}]\,w^{s_1}(\boldsymbol{p}_1) \quad (195)$$

This clear cut dependence on the e.m. coupling constant α becomes transformed into parameters that allow the familiar physical comparison with entities known from microphysics and quantum mechanics. The coupling constant $e^2/\hbar c$ gets dismantled into the classical electron radius $r_0 = e^2/4\pi mc^2$ and the superfluous $\hbar c$ accomodated under the square roots to combine with each $|k_0|$ to give $\omega = \hbar c|k_0|$. The amplitude thus gets displayed as [82] (Schweber 1962,488)

$$R_{ab} = \delta^{(4)}(p_2 + k_2 - p_1 - k_1)\frac{r_0}{2\pi i}\frac{m^2}{\sqrt{E_1 E_2 \omega_1 \omega_2}}\tilde{w}^{s_2}(\boldsymbol{p}_2)\,[\text{spin-polarization terms}]\,w^{s_1}(\boldsymbol{p}_1) \quad (196)$$

This straightforwardly leads to the well known *Klein-Nishina* formula (Schweber 1962,491)

$$d\sigma = r_0^2\left(\frac{\omega_2}{\omega_1}\right)^2 F d\Omega \quad (197)$$

with the classical *Thomson* formula as a non-relativistic limit

$$d\sigma_{nr} = r_0^2\, cos^2\Theta\, d\Omega \quad (198)$$

Every hint to flat space as the generator of spinors has volatilized in favor of a description in terms of geometrical properties of classical particles that live *in* space.

C.3.4. The anomalous magnetic moment of the electron

A similar case is given by the anomalous magnetic moment of the electron $\frac{e\hbar}{2mc}$. The dependence on $\hbar$ seems to be a clear hint to the quantum mechanical origin of the moment. So let us see what happens in the calculation within the franework of QED.

The relevant amplitude describing the scattering of an electron in an external e.m. field is given by (Schweber 1962,543):

$$R = +2\pi ie\tilde{u}(p_2)(\gamma_\nu + \Lambda_{C\nu}(p_2, p_1))u(p_1)a^\nu(p_2 - p_1) \quad (199)$$

where the first term is the contribution without radiative corrections and $\Lambda_{C\nu}$ represents the radiative corrections after renomalization. This may be cast into the form [83]

$$R = +2\pi ie\tilde{u}(p_2)\left\{F(k^2)\gamma_\nu - \frac{i}{2m}k^\mu\sigma_{\nu\mu}\right\}u(p_1)a^\nu(p_2 - p_1) \quad (200)$$

with $k_\mu = p_{2\mu} - p_{1\mu}$.

How does the magnetic moment of the electron enter the game? Let us go back to the Dirac equation. The electromagnetic properties of a Dirac particle are best exhibited by a transformation to a Foldy-Wouthuysen representation. However: *"In the presence of interaction, the generator for the transformation can only be obtained as a power series expansion in powers of the Compton wave length h/mc of the particle. The transfomed Hamiltonian is therefore likewise obtainable only in a power series in the same parameter"* (Schweber 1962,102). This expansion pops up an interaction term of the Dirac particle with a magnetic field:

$$-\frac{e\hbar}{2mc}\beta\vec{\boldsymbol{\sigma}}\cdot\vec{\boldsymbol{H}} \quad (201)$$

signalling an *anomalous magnetic moment* of one Bohr magneton $\frac{e\hbar}{2mc}$ [84]. This is how $\hbar$ enters the scene.

[82] Since working with units $\hbar = c = 1$ $\omega = |\boldsymbol{k}|$ denotes the energy of the photon.

[83] This is the most general form of the matrix element and follows from the relativistic and gauge invariance of the S-matrix formalism and from t+he assumption that the external field is weak so that- only terms linear in the external field need be considered (Schweber 1962,543).

[84] Gauss units

Foldy conceived the most general Dirac equation in powers of k^{2n} with coefficients ϵ_n and μ_n, where ϵ_0 is the static charge and μ_0 the static anomalous magnetic moment. This leads to an amplitude (Schweber 1962,544)

$$R_F = -2\pi i\tilde{u}(p_2)\sum_{n=0}^{\infty}[\epsilon_n k^{2n}\gamma_\nu - ik^\mu\sigma_{\nu\mu}\mu_n k^{2n}]u(p_1)a^\nu(p_2-p_1) \tag{202}$$

By comparison we see that the radiative corrections of QED lead to a charge distribution $-eF(k^2)$ and an anomalous magnetic moment distribution $\frac{-e}{2m}G(k^2)$. Expanding $G(k^2)$ and $F(k^2)$ [85] in powers of k^2 we see that $\frac{-e}{2m}G(0) = \mu_0$, μ_0 denoting the Foldy coefficient referring to the anomalous magnetic moment in lowest order. Again, nowhere in this comparison $\hbar$ enters the stage.

The decisive clue with respect to our question is provided by the observation that the magnetic moment μ_0 is multiplied with the associated matrix $k^\mu\sigma_{\nu\mu}$ of the covariant wave vector k_μ of the photon. Like in the case of the Rydberg constant, it is up to our convention in what terms we *interprete* the result. The QED calculation gives the simple result $G(0) = \alpha/2\pi$, i.e. in terms of a parameter referring to flat space with no $\hbar$ present. $\hbar$ enters post festo when we translate the covariant photon wave vector k_μ into the contravariant entity p_μ by borrowing $p_\mu = \hbar k_\mu$ from Quantum Mechanics [86]. Our wish for *interpretation* whence makes us express the result of QED $(1+\alpha/2\pi)$ in terms of quantum mechanical *Bohr magnetons* (Schweber 1962, 544).

These considerations apply as well to the Lamb shift as to other results of QED or likewise the Dirac equation. They show that no $\hbar$ is present in QED.

C.3.5. Rutherford scattering

Our last exmple Rutherford scattering, is a beautiful example for generating the dependence on α not by the additional factor $\sqrt{\hbar c}$ of photon lines but by multiplication with the Cartan associated matrix of the e.m. potential $\not{a}$ in momentum space.

The differential cross section for the scattering of an electron on an external field according to QED is given by (Schweber 1962,456):

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2}{4p^2v^2sin^4\frac{\Theta}{2}}(1 - v^2 sin^2\frac{1}{2}\Theta) \tag{204}$$

with v the velocity of the incoming particle (c=1) and Θ the scattering angle. This *Rutherford* formula with the spin related additional factor $(1 - v^2sin^2\frac{1}{2}\Theta)$ clearly displays an α-dependence, but not with first power of α as expected for a first order process.

It is worthwile to have a glance on how the coupling constant α comes about in this case. The amplitude of the respective Feynman diagram to order $n=1$ is given by

$$M_e^{(1)} = \left(\frac{ie}{\hbar c}\right)\left(\frac{m^2}{E(\boldsymbol{p}_1)E(\boldsymbol{p}_2)}\right)^{1/2} 2\pi\tilde{u}_{s_2}(\boldsymbol{p}_2)\ \not{a}(p_2-p_1)u_{s_1}(\boldsymbol{p}_1) \tag{205}$$

If the external e.m. field is the Coulomb field of a nucleus of charge $+Ze$ then $A^0 = Ze/(4\pi r)$ and the associated matrix of its Fourier transform is

$$\not{a}(q) = \frac{Ze}{4\pi}\frac{1}{2\pi^2\boldsymbol{q}^2}\delta(q_0)\gamma^0 \tag{206}$$

The transition amplitude therefore is given by

$$M_e^{(1)} = \frac{Ze^2 i}{4\pi^2\hbar c}\left(\frac{m^2}{E(\boldsymbol{p}_1)E(\boldsymbol{p}_2)}\right)^{1/2}\tilde{u}_{s_2}(\boldsymbol{p}_2)\frac{1}{|\boldsymbol{p}_2-\boldsymbol{p}_1|^2}\gamma^0 u_{s_1}(\boldsymbol{p}_1)\delta(E2-E1) \tag{207}$$

[85] $F(k^2)$ gives a lengthy expression which approaches 1 and

$$G(k^2) = \frac{\alpha}{2\pi}\frac{2\Theta}{sin2\Theta}; \qquad sin^2\Theta = \frac{k^2}{4m^2} \tag{203}$$

which approaches $\frac{\alpha}{2\pi}$ for $k^2 \to 0$.

[86] For easier reading we skipped the metric $g_{\mu\nu}$ which would have to become applied when translating from covariant to contravariant entities and vice versa. We thus adapt to Quantum Mechanics which operates in 3-dim space where the metric is obsolete. And so we loosely use $p_\mu = \hbar k_\mu$ to indicate the transition from covariant to contravariant entities. Consequently this should have been written as: $p^\mu = g^{\mu\nu}\hbar k_\mu$.

where $E_2 = E_1 = E$ is the energy of the electron (Schweber 1962,453/454). Thus already in the first order amplitude a factor $\alpha = e^2/\hbar c$ gets established. The analysis of the Feynman amplitudes in the last section but showed that the power expansion of the S-matrix delivered a factor $(-ie/\hbar c)$ per knot and that the critical ratio $e^2/\hbar c$ of the e.m. coupling constant became established by multiplication not with e but with $\sqrt{\hbar c}$ per knot from the photon contribution so that at every knot the e.m. coupling constant becomes manifest *after* taking the square of the amplitude.

Rutherford scattering exposes that the $\sqrt{\hbar c}$ contribution of the photon gets replaced by the associated matrix $\not{a}(q)\,(\boldsymbol{q} = \boldsymbol{p_2} - \boldsymbol{p_1})$ of the Fourier transformed e.m. potential which shows no reference to $\hbar$ whatever. The appearance of the associated matrix is the guarantor for the disappearance of the $\hbar$ reference. What seemed to be a 1^{st} order approximation now corresponds to the 2nd order Compton scattering with one particle having infinite mass. That is the reason why *Rutherford* scattering shows the same α-dependence as the 2nd order *Compton* scattering (sect.C.3.3) does.

D. The nature of fundamental constants

We will recall the role of the constants κ and $\hbar$ that make them to be considered fundamental. And we will recall the distinct nature of constants like the *velocity of light* c and the *elementary charge* e. But let us comment on the meaning of λ, the cosmological constant.

xxxxx

D.1. The fundamental constant λ

It is instructive to note that the factual value of the constant λ is incredibly small. In all practical numerical calculations the λ-term may be neglected and eq.(3) may be replaced by

$$G_{\mu\nu} = 0. \tag{208}$$

In spite of the disappearance of λ in further calculations Eddington's derivation of the Einstein field equation shows that λ plays a crucial role. For Eddington λ was irreplaceable for systematic reasons. Chandrasekhar in his 1983 laudation (Chandrasekhar 1983,39) quoted him [87] : *to set* $\Lambda = 0$ *is to knock the bottom out of space* (Chandrasekhar 1983,39).

λ today is believed to provide a measure for the contribution of dark energy in the cosmos. The observational results, so a 2016 Chandra bulletin (http://chandra.si.edu/photo/2016/clusters/, Release Date April 28,2016), *"support the idea that dark energy is best explained by the cosmological constant"* ... The results *"confirm earlier studies that the amount of dark energy has not changed over billions of years."*

From the derivation of Eddington this behaviour might have been expected. What we call *dark energy* according to his derivation is reflecting the feature that physicists approach the world by means of measuring. λ serves to identify the metric (used in our observations and which reflects our familiar units) with the Einstein tensor $G_{\mu\nu}$ (the units of which are unknown). Hence the existence of λ is important but not its value. Eddington compares this circumstance with the enormous size of the entropy, whose absolute value is of no importance but whose existence as an entity with a finite value allowing for definite comparison makes up its systematic importance. (Edd 1923,TBD).

For empty space Edddington derives:

$$-\,ds^2 = 3/\lambda \tag{209}$$

This means the quadric of curvature is a sphere of radius $\sqrt{3/\lambda}$ and the radius of curvature in every direction and at every point in empty space has the constant length $\sqrt{3/\lambda}$. Conversely if the directed radius of curvature in empty space is homogeneous and isotropic Einstein's law will hold.

This means: The length of a specified material structure bears a constant ratio to the radius of curvature of the world at the place and in the direction in which it gets measured (Edd 1923,153).

[87] Not without expressing his doubt: *In any event, it is clear that no serious student of relativity is likely to subscribe to Eddington's view that 'to set* $\Lambda = 0$ *is to knock the bottom out of space'.* (Chandrasekhar 1983,39)

D.2. The fundamental constants κ and $\hbar$

D.2.1. Introducing covariant entities as standard opened up a wide technical window of accuracy

The quantum-mechanical identification of the contravariant energy with the covariant *frequency* opened up a completely new technical window for the accuracy of measurement [88]. It was the start for discarding material prototypes of measuring units in favor of the use of fundamental constants.

Taking a covariant entity, the wave vector, as the base of measurement in the 1970's led to an explosive extension of the outreach of accuracy down to microscopic scales.

The essential role of measuring frequencies in quantum metrology (Cook 1972, Cook 1975, Petley 1983) is given by the fact that multiplication with $\hbar$ delivers the energy of the system, $h\nu$. This makes up the pivotal role of the gyromagnetic ratio $2\mu_P/h$ of the proton for the determination of the magnetic field $h\nu = 2\mu_P B$ and of the Josephson constant $2e/h$ for the determination of the electric field, $n \times h\nu = 2eV$.

D.2.2. Contravariant measuring entities need to be identified with covariant space variables

Eddington's conjecture encompasses a theory of measurement. In this framing fundamental constants are featuring the identification of the contravariant measuring entities of classical physics with the covariant variables of the respective space concept. This signifies one of these dimensions to become redundant.

The constants which identify contravariant measuring entities with covariant space variables are:

- κ, the Newtonian gravitational constant, that makes the measuring entities of classical physics compiled in the energy-stress tensor $T^{\mu\nu}$ to be identified with the curvatures of the Riemannian space concept compiled in the Einstein tensor $G_{\mu\nu}$. Given in $[cmg^{-1}]$ the Newtonian constant makes the inertial mass given in $[g]$ to be identified with the heavy mass given in $[cm]$.
- $\hbar$, Planck's action quantum, that makes the energy-momentum p^μ to be identified with the wave vector k^μ [89]. Given in $[erg \cdot s]$ it allows to identify the contravariant entity energy $[erg]$ with the covariant entity *frequency* $[s^{-1}]$ of the wave vector.

Mass historically has been introduced as *heavy* mass with the dimension $[cm]$ and as *inertial* mass with dimension $[g]$, because space and matter at the time of Newton were believed to be mutually independent. The experimental finding that both kinds of masses are equivalent found an early expression by the Newtonian gravitational constant κ with the dimension $[cm/g]$ signifying that one of these two dimensions is redundant. Because it corrects for the historically introduced distinction this constant has a fixed value which makes it to become a *fundamental* constant.

Plancks action quantum $\hbar$ by identifying the contravariant entity *energy* with the wave variable *frequency* signifies that particles and waves are mutually conditioning each other making one of these dimensions, energy or frequency, to become redundant. Also this constant is considered fundamental because having a value that is historically fixed.

D.2.3. Featuring the transition from a logical exclusion principle to mutual conditioning

Plancks action quantum as well as Newton's gravitational constant signal the abolition of the government of a classical european philosophy featuring an exclusion principle:

- space $[cm]$ and matter $[g]$ in Newtonian philosophy were thought to be mutually exclusive and consequently equipped with dimensions that appeared to be independent. κ $[cm/g]$ signals the scar where the intimate relation between these two dimensions got restored. In general relativity the Newtonian constant κ with the dimension $[cm/g]$ mediates the

[88] There is a grave reason to prefer a measurement based on frequencies against one based on wave lengths:

> "...there is no doubt that the more fundamental standard is one of frequency. ... Frequencies can be established and compared at a single site without regard to geometry or extension in space, whereas the definition of wavelength depends on geometrical circumstances"(Cook 1972, 487)

[89] separately for each non-relativistic component

necessary correction of the historical Newtonian division to regard matter $[g]$ and space $[cm]$ as two entities seemingly completely independent of each other. The Einstein field equations describe how space and matter are mutually conditioning each other: a distribution of matter is curving space and the curvature is determining the motion and hence the distribution of matter.

- particles $[erg]$ and waves $[s^{-1}]$ in a classical view seemed to be fundamentally distinct: particles could be thought of being localizable, whereas waves by definition could not be localized. This was the base of the century long dispute between the Newtonian corpuscular theory and the Fresnel wave theory of light. $\hbar$ $[erg/s^{-1}]$ signals the scar where an intimate relation between these two aspects got restored. The Planck constant $\hbar$ $[erg]/s^{-1}$ mediates the necessary correction of a historical European division that regarded particles represented by p^{μ} $[erg]$ as completely independend of waves represented by the covariant wave vector k_{μ} $[s^{-1}]$. In Quantum Mechanics the objects are neither waves nor particles. Instead both these specifications are conditioning each other making particles to behave as waves and waves becoming represented by quanta. Quantum Mechanics succeeded to describe waves and particles as two different aspects of the same object.

Both universal constants have fixed values because they undo the *historical decision* to consider the respective dimensions as mutually exclusive. Their value hence is historically fixed. There is not any need nor any possibility to calculate these constants within the theoretical framework of physics.

D.2.4. $\hbar$ is of relativistic nature though Quantum Mechanics is a non-relativistic theory

Relativistic theories are living in spaces endowed with a pseudo-euclidean metric allowing to operate with a preferred coordinate identified with *time* called $x_0 = ict$. Quantum Mechanics genuinely is a *non-relativistic* theory. The concept of Quantum Mechanics relies on the representation of measurable variables by hermitian operators. An imaginary time does not allow for a hermitian operator representation. In the framework of Quantum Mechanics hence space and time are not linkable within a common metric. Quantum Mechanics is a genuinely non-relativistic theory.

It is remarkable that although $\hbar$ emerged in the framework of a non-relativistic theory, it nevertheless mediates relativistic relations. The non-relativistic identifications (see p.25) of Quantum Mechanics relating to $\hbar$ easily allow for a relativistic notation: $p_{\mu} = \hbar k_{\mu}$, $A_{\mu} = \hbar a_{\mu}$, $m_0 = \hbar \mu$. This series is complemented by the identification $e^2 = \alpha_{em}\hbar$. e^2 is referring to Maxwells electromagnetism, a genuinely relativistic theory, and α_{em} is belonging to the Standard Model which as well describes a relativistic theory.

This relativistic nature of $\hbar$ is the condition to give it such a preeminent role in the determination of the SI units by the BIPM.

D.3. The fundamental constants c and e

D.3.1. The fundamental constant c: correcting for a historical misconception

It is worthwile to shed light on the constant c. This constant should not be confused with the velocity of light. Only in SRT the velocity of light takes the constant value c. In General Relativity the velocity of light is different at distinct locations and even at the same location it may be different for distinct directions. (Edd 1923,93)

By introducing the imaginary coordinate $x^0 = ict$ SRT aimed for two achievements.

- The fundamental constant c $[cms^{-1}]$ corrects for the Newtonian judgement to posit space and time as two independent pilars of perception equipped with distinct dimensions $[cm]$ and $[s]$. In the framework of SRT time is given a position highly symmetrical to space making the invariant length element ressemble the quadratic form of flat space in four dimensions underlined by eventually setting $c = 1$.

- Making time an imaginary coordinate was the only way to allow to identify the electric and magnetic field with the polar and axial components of the bivector by maintaining them to be measurable entities. (see Cartan 1938,132)

Whereas the constant c may be set to 1 to achieve simplifications in representation the imaginary character of time may not be discarded without running into contradictions.

The constant c corrects for a historical misconception taking time and space to be independent categories. The value of c hence is historically fixed and does not require for any physical determination. In contrast to κ and $\hbar$ c does not identify contravariant with covariant variables but operates in the contravariant sector alone. This allows to consider the 4-dim space of SRT as a representative of flat space endowed with a pseudo-euclidean metric.

The primary goal of Lorentz transformations in our opinion is not to keep the velocity of light constant in any reference system. [90] Instead they guarantee the correction c for the misconception of taking time as fundamentally independent of space being the same in every reference frame. This then coincides with the velocity of light.

D.3.2. The electric charge e: signifyer of the antisymmetric sector of the space concept

There are similarities between the electric charge e and the two constants κ and $\hbar$. All three are mediating the identification of contravariant measuring entities with covariant space variables. But the siuation concerning the electric charge is a little more subtle.

In the case of electric charge it is the electric current J^μ which gets identified with the divergence of the electromagnetic field $F_{\mu\nu}$

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = J^\mu \tag{210}$$

Though looking similar to the identification of the energy-stress tensor $T^{\mu\nu}$ with the Einstein tensor $G_{\mu\nu}$

$$G^{\mu\nu} - 1/2g^{\mu\nu}G = 8\pi\kappa T^{\mu\nu} \tag{211}$$

there is nevertheless a fundamental difference: the electromagnetic field $F_{\mu\nu}$ belongs to the *antisymmetric* sector of space whereas $G_{\mu\nu}$ represents the *symmetric* sector. And we note: e, the electric charge, which serves to identify the electric current with the divergence of the bivector of flat space is not displayed explicitly.

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The procedure of classical electrodynamics to not display the charge explicitly in eq.(210) means to give the field $F_{\mu\nu}$ the dimension $[charge/cm^2]$ making the electric charge to become a signifier of the antisymmetric sector of the flat space concept. The density ρ then automatically denotes a charge density. This procedure though consistent hides the origin of the e.m. field $F_{\mu\nu}$ as the antisymmetric twin of the symmetric Einstein tensor $G_{\mu\nu}$ both deriving as the trace of the Riemann-Christoffel tensor $B^\lambda_{\mu\nu\sigma}$. (Edd 1923,198,223)

Contrary to the fundamental constants κ and $\hbar$ signifying that a dimension that had been introduced historically turned out to be redundant the electric charge e does not stand for the redundancy of whatever dimension. The electric charge takes the role of a *signifyer* signalling that we operate in the antisymmetric sector of the space concept. This makes up the autonomous meaning of the dimension $[charge]$.

D.3.3. The electric charge is the quantum mechanical *equivalent of the e.m. coupling constant α_{em}*

The weak and the strong interactions of the Standard Model are mediated by short-range fields. Because of the long-range character of the e.m. interaction which made parts of the mediating field perceptible even for human eyes the bivector of QED became represented by a classical electromagnetic field $F_{\mu\nu}$. Maxwells theory of electromagnetism is an early instance of the interactions that genuinely are the object of the Standard Model.

The electric and magnetic fields became successfully identified with the polar and axial components of the bivector $F_{\mu\nu}$ (see sect.4, p.22) after a pseudo-euclidean metric had been imprinted on flat space (see sect.4.2.1, p.23).

By identifying the covariant variable of the space concept, viz. the bivector $F_{\mu\nu}$, with the contravariant measuring entity, the current density J^μ, the electric charge in Maxwells's electromagnetism seems to adopt a role similar to the one the fundamental constant κ adopted in General Relativity and $\hbar$ in Quantum Mechanics.

But there is another aspect making the nature of the *electric charge* completely distinct from these two constants.

These latter denote each a rollback of a historical decision. The Newtonian constant κ $[cm/g]$ is curing the historical split of the heavy mass $[cm]$ and the inertial mass $[g]$ into distinct dimensions one of which turned out to be superfluous. Similarly Plancks quantum $\hbar$ $[erg \cdot s]$ is curing the split between energy $[erg]$ and frequency s^{-1} making one of them to become superfluous.

[90] Why should it be constant? In General Relativity the velocity of light may be different at different locations and even at the same location it may be different in different directions.

The electric charge historically has got another role. The electric and magnetic field have got assigned the dimension $[chargecm^{-2}$, to make the divergence a charge density. This means $\frac{1}{e}F_{\mu\nu}$ would denote the antisymmetric twin of the metric $g_{\mu\nu}$. Electric charge has become a signifyer denoting the electric and magnetic field to represent the antisymmetric sector of the space concept.

But where does the electric charge get its value from?

In QED α_{em} takes on the value $\alpha_{em} = e^2/(\hbar \cdot c)$. Putting $c = 1$, this relation reads

$$e^2 = \alpha_{em}\hbar \tag{212}$$

Comparison with the set of identifications characterizing Quantum Mechanics (p.25) shows the electric charge e^2 to be the contravariant equivalent of the covariant coupling constant α_{em}, as could be expected.

According to Wyler (1968) α_{em} is conjectured to have a fixed value $\alpha_{em} \propto 1/137$ determined by group theoretical relations intrinsic to flat space. The electric charge hence is not a fundamental constant because signifying a historical decision but because representing a geometric property of flat space like do the coupling constants of the other interactions.

The mediating constant of Maxwell's electromagnetism appears to be mediated itself by $\hbar$ referring to the framework of Quantum Mechanics. Quantum Mechanics hence seems to take the role of a mediator between Maxwell's electromagnetism and QED.

Contrary to the fundamental constants κ and $\hbar$, that signify that a dimension that had been introduced historically turned out to be redundant the electric charge e does not stand for the redundancy of whatever dimension. The electric charge takes the role of a signifyer signalling that we are in the antisymmetric sector of the space concept and its constant value represents a geometric property of flat space when taken to be complex.

D.4. The coupling constants of elementary particle physics

D.4.1. The Cartan invariant makes the identification of contravariant with covariant entities obsolete

In case of *elementary particle physics* an identification of contravariant measuring entities (vectors) with covariant space variables (spinors) is obsolete.

There is a simple reason for this: The new Cartan invariant $\xi^{T'} C \underset{(p)}{X} \xi$ is built up from covariant (spinor) and contravariant (vector) entries. Identifying fermions with spinors and bosons with the associated matrices of multivectors, the Yukawa form of the invariant allows for a dynamical interpretation: a fermion gets transmuted to become another fermion mediated by a boson. Several such transmutations are taking part under the roof of the same invariant. This allows to get experimental results in terms of conditional probabilities. No identification of contravariant with covariant entities is needed.

In case of the e.m. interaction the new covariant coupling constant α_{em} reappears in Maxwell's electromagnetism as the contravariant electric charge $e^2/c = \alpha_{em}\hbar$. e serves to identify the divergence of the bivector field $F_{\mu\nu}$ with the current density J^μ.

In terms of QED this current density is made of two spinors, $\tilde{\psi}\gamma^\mu\psi$. Whereas the fields in General Relativity $G_{\mu\nu}$ have a dimension $[cm^{-2}]$ the electric charge for historical reasons has been integrated within the dimension of the e.m. field to become $[charge\, cm^{-2}]$. The e.m. identification $\partial_\nu F^{\mu\nu} = J^\mu$ then operates with a dimension $[charge\, cm^{-3}]$ without exposing the charge explicitly. [91]

D.4.2. The coupling constants denote geometrical features of flat space

Another type of constant is entering the theatre. Since the different types of interactions result from representations of the Cartan invariant in different dimensions, the ratio of the group volumes of the respective spaces is expected to provide a measure for their relative coupling constants.

Without knowing Cartan's spinor theory and its application to the Standard Model Wyler in 1968 conjectured the value of the electromagnetic coupling constant to be given as the group-theoretical ratio of volumes of flat hyperspaces allowing him to calculate the famous value $1/137$ for the e.m. coupling constant. His method later became extended to calculate the coupling constants α_{weak}, (TBD) and α_{strong} (TBD) of the weak and strong interactions.

[91] There is a hiatus in our exposition up to now. We did not investigate why the electric charge appears squared in the relation $e^2/c = \alpha_{em}\hbar$.

The coupling constants of elementary particle physics, the most famous one being the fine structure constant $\alpha_{em} \propto 1/137$, are conjectured to denote ratios of group volumes of flat space (Wyler 1968,...TBD). This is supported by the finding that the distinct interactions of elementary particle physics represent the Cartan invariant in different dimensions of complex flat space.

The value of these constants whence is essentially determined by geometrical relations of flat space as first calculated by Hua (TBD) and used by Wyler (1968,1969,1972,TBD).

E. Towards experimental verification

E.1. Basic postulates of the theory of elementary particles are recovered

Basic postulates of the theory of elementary particles are recovered:

- the term in the spinor theory of Cartan which we identify with an interaction naturally describes the occurrence of parity violation under certain conditions, e.g. for $\nu = 2$ and $p = 1$. This parity violation had to be introduced ad hoc in the Standard Model.
- For $\nu = 4$ the equal number of components (eight) of the isotropic vector and the two semi-spinors leads to the phenomenon of *triality* described extensively by Cartan. We conjecture this triality to be the reason for the three generations observed for elementary particles [92].
- the existence of two classes of particles, right-handed ones and left-handed ones
- the existence of a new type of invariants providing for the interactions,
- the TCP theorem,
- the Fermi statistics,
- the parity violation in weak interactions,
- the confluence of e.m. and weak interactions into an electro-weak

They may be easily *derived* within the framework of the spinor theory of flat space.

E.2. the quest for unification TBD

E.2.1. Spinor 16^+ seems to be the answer to the quest for unification

The quest for unification has led to probe several higher symmetries. Wilczek (2006) underlines that to allow for a more extensive symmetry the quarks and leptons must furnish the material for building unified structures that remain coherent under the extended symmetry. He notes:

> ”One particular unified symmetry passes this test with flying colors. Although the smallest simple group into which $SU(3) \otimes SU(2) \otimes U(1)$ could possibly fit is $SU(5)$ - it fits all the fermions of a single family into two representations ($\mathbf{10} + \mathbf{5}$) and the hypercharges click into place - a larger symmetry group, $SO(10)$ fits these and one additional $SU(2) \otimes SU(2) \otimes U(1)$ singlet particle into a single representation (the spinor $\mathbf{16}$). All 15 quarks and leptons appear on the same footing and the additional particle, which has the quantum numbers of a right-handed neutrino, is quite welcome: it plays a crucial role in the attractive ’seesaw’ model for neutrino masses.”(Wilczek 2006,242 (written 2003))

[92] The objections of Zee against the derivation of the three generations from triality in our opinion don’t apply. They are based on the group theory of *unitary* transformations which in general have no spin representations (Zee,2016)

It turns out that spin(16) viz. the $1/2 \times 2^{\nu}$-dimensional semi-spinor representation related to $\nu = 5$ in fact does decompose into all the representations that are required for the Standard Model and its appropriate extension (sect.F.3.3,p.F.3.3): $16^{+} \to 1 \oplus 10 \oplus 5^{*}$. So it contains what was expected and hoped for already by using $SU(5)$.

The spinor representations of $SO(2\nu)$ indeed have the exponential dependence characterizing its dimension to be $dim(SO(2\nu)) = 2^{\nu}$, as we know it from Cartan. For the space dimension $2\nu = 10$ being even this leads to an irreducible representation in form of semi-spinors with dimension $2^{\nu-1}$ (sect.3.3.2, p.16). Thus for $\nu = 5$ this results in the now famous spinor 16^{+}, which assembles all particles including the antineutrino in one representation, all of them with the correct quantum numbers.

Wilczek concludes:

> ”Where before we had a piecemeal accomodation of the observed particles, now we have a marvellous correspondence between reality and a unique, ideal mathematical object.”(Wilczek 2006,242)

E.2.2. The antineutrino as an additional benefit of spin(16)

The discovery provided for an additional benefit. The *mysterious intruder*, an ”additional” singlet under $SU(5)$ turns out to be the right handed neutrino, i.e. the *long lost antineutrino* field. It is a singlet under $SU(5)$ and thus the more under $SU(3) \times SU(2) \times U(1)$. (Zee 2016,552)

It is most intriguing that the extra field in the sixteen-dimensional representation of $SO(10)$ turns out to have precisely the right properties to be associated with a neutrino spinning right-handedly. [93]

E.3. Tentative identification of the interactions

The series of electromagnetic, electroweak and strong interactions could naively be identified with the series of dimensions ν and their respective unitary transformations $SU(\nu)$. The spinors in dimension ν have 2^{ν} components and could then be:

- $\nu = 1$ e.m. interaction (e_R^-, e_R^+)
- $\nu = 2$ el.weak interaction (e^-L, ν_L, u_L, d_L)
- $\nu = 3$ strong interaction $(e^-L, \nu_L, u_L^{rgb}, d_L^{rgb})$

But this obviously is to simple an identification, since the left- or right handed fundamental particles should be the components of semispinors, which have $2^{\nu-1}$ components only, and moreover the charge conjugates are missing in this too simple scheme.

F. How can experiments/observations report the structure of the space concept?

F.1. The crucial question of theory building

We are facing the crucial question of physical theory building:

- is the gravitational field an object of Nature that our measuring appliances detect in Nature and which we are able to describe by the metric of Riemannian space? Are the leptons and quarks objects of Nature, that our measuring appliances detect and which we happily can describe as spinors? How then could it occur that Nature is mimicking the structure of our space concepts down to such a level of ramification?
- or do physicists adopt a space concept that allows them to encode the condition of the possiility to measure and theory and experiment trace the physiognomy of the adopted space concept ?

[93] ”This Weyl field does not participate in the strong, weak, and electromagnetic interactions. In plain English, he is a lepton with no electric charge, and is not involved in the known weak interaction. Thus we identify the mysterious 1 as the ”long lost” antineutrino field ν_L^c. This guy doesn't listen to any gauge bosons, known or unknown to experiments. Even the gauge bosons of $SU(5)$ don't know about him.
We are using a convention in which all fermion fields are left handed and hence we have written ν_L^c. By a conjugate transformation this is equivalent to the right-handed neutrino field, which was missing from the $SU(3 \otimes SU(2) \otimes U(1))$ theory.
Why have experimentalists not seen it? The natural explanation is that this field is endowed with a large Majorana mass.”
(Zee 2016,551).

We try to convince our readers that there is no way to avoid the 2nd option, though it is contrary to traditional belief.

This 2^{nd} option says: Men by measuring encounter nothing but themselves and their obsession to measure.

The objectivity of the results, viz. the independence of the results from *who* is performing the measurement then has its roots not in the autonomous existence of something called Nature but is rooted in the autonomous framing established by the underlying space concept equipped with the respective condition of the possibility to measure.

The consistency of the theory is borrowed from the consistency of the space concept. The fact that theory and experiment work hand in hand is effected by something like the *invisible hand* of the consistency of the space concept.

We display in detail three examples of how this invisible hand of the space concept is working practically:

- Maxwell's electromagnetism as emerging from the day-to-day experimental and theoretical efforts of physicists
- elementary particle physics (Standard Model) as based on a new aspect of traditional flat space
- General Relativity as a new theory based on a radically new concept of space

The space concept of General Relativity is *Riemannian* space. All the other branches of physics rely on *flat* space, adopting their *condition of the possibility to measure* by making use of specific extensions of this concept.

F.2. Maxwell's theory of electromagnetism

F.2.1. Faraday, Gauss, Ampère, Maxwell

Maxwell's four homogeneous equations were composed of Faraday's law of induction [94]:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{213}$$

and of Gauss's law for magnetism

$$\nabla \cdot \vec{B} = 0 \tag{214}$$

Maxwell's four inhomogeneous equations were composed of Maxwell-Ampere's law

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{215}$$

and Gauss's law for electricity

$$\nabla \cdot \vec{E} = \rho / \epsilon_0 \,. \tag{216}$$

Note that $\mu_0 \epsilon_0 c^2 = 1$.

F.2.2. The introduction of a displacement current by Maxwell

In Ampere's original equations Maxwell had to posit an extra current $\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ in eq.(215) called the *displacement current*. It was needed in order to eplain magnetic fields that are produced by changing electric fields and to guarantee consistency between Ampere's circuital law for the magnetic field and the continuity equation for electric charge.

The current leaving a volume must equal the rate of decrease of charge in a volume. Put in a differential form this is the continuity relation

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \tag{217}$$

Ampere's law in its original form states

$$\nabla \times \vec{B} = \mu_0 \vec{j} \tag{218}$$

which implies the divergence of the current to vanish by virtue of the identity

$$\nabla \cdot \nabla \times \vec{A} = 0 \tag{219}$$

[94] we use SI units

$\vec{A}$ being any vector. This contradicts the continuity equation. Adding the displacement term

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{220}$$

we get

$$\nabla \vec{j} + \epsilon_0 \frac{\partial \nabla \vec{E}}{\partial t} = 0 \tag{221}$$

implying by Gauss's law the continuity equation

$$\nabla \vec{j} = -\frac{\partial \rho}{\partial t}\,. \tag{222}$$

The introduction of the displacement current directly implies the high symmetry of Maxwell's equations in empty space ($\vec{j} = \rho = 0$)

$$\nabla \cdot \vec{E} = 0 \tag{223}$$

$$\nabla \cdot \vec{B} = 0 \tag{224}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{225}$$

$$\nabla \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} \tag{226}$$

which implies the wave equations of e.m. fields. Taking the curl of eq.(225)

$$\nabla \times \nabla \times \vec{E} = -\frac{\partial (\nabla \times \vec{B})}{\partial t} \tag{227}$$

and substituting eq.(226) leads to

$$\nabla \times \nabla \times \vec{E} = -\frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} \tag{228}$$

With $\nabla \times \nabla \times \vec{E} = \nabla \cdot (\nabla \vec{E}) - \nabla^2 \vec{E})$ we get with eq.(223) the wave equation of a plane wave for the electric field

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} \tag{229}$$

The high symmetry guarantees a similar wave equation for the magnetic field. Both wave equations are mirroring the defining characteristics of a flat space endowed with a pseudo-euclidean metric.

To grasp the intrinsic necessity of a *pseudo-euclidean* metric we will have a look at the bivector $F_{\mu\nu}$ of flat space, which gets identified with the electromagnetic field tensor $F^{em}_{\mu\nu}$.

F.2.3. The seesaw of the historical development

Using the displacement current in this derivation by now is generally accepted as a historical landmark in physics because it allowed uniting electricity, magnetism and optics into one single unified theory. The displacement current term is seen as a crucial addition completing Maxwell's equations necessary to explain many phenomena, most particularly the existence of electromagnetic waves. [95]

Maxwell's physical derivation using a sea of molecular vortices is unrelated to the modern day derivation [96] which has its merits in free space. Not only that for this modern derivation a displacement current should exist in free space. By featuring the wave equation it characterizes this free space to be a flat space constituted by the foundational quadratic equation endowed with a pseudo-euclidean metric.

[95] The introduction of the displacement current is based on consistency between Ampere's circuital law for the magnetic field and the continuity equation for electric charge. It allows to predict correct magnetic fields in regions where no free current flows; it allows the prediction of wave propagation of electromagnetic fields and the conservation of electric charge in cases where charge density is time-varying.

[96] https://en.wikipedia.org/wiki/Displacement_current#Wave_propagation, history and interpretation. (last call 31.Dec.2020)

Guided by consistency relations the experimental and theoretical efforts turn out to trace the physiognomy of flat space. Not only the wave equations display a pseudo-euclidean metric but already the identification of the electromagnetic tensor with the bivector of flat space to be consistent demand this metric.

This metric then achieves Maxwells four inhomogeneous equations to become the relativistic identification eq.(70) of the covariant variables of the space concept compiled in $F^{em}_{\mu\nu}$ with the contravariant measuring entities compiled in J^{μ} provided the two restrictive conditions eq.(63) and eq.(67) set by the bivector are met.

Both intermezzi, introducing the displacement current and imposing on space a pseudo-euclidean structure, vividly document the flexibility with which theoretical description and experimental conceptualization are conditioning each other, driven by and effecting a consistency of the space concept.

F.2.4. The emergence of SRT

Recognizing the exchange of signals by means of plane waves of electromagnetic light as one of the prominent means to built a *consistent* apparatus of measuring required the pseudo-euclidean metric to become one of the basic fondaments of space concepts needed for measuring in physics. Special relativity (SRT) makes the flat space concept developed in the framework of Maxwell's electromagnetism be mandatory for mechanics as well. SRT implied to adapt the equations of classical mechanics accordingly by integrating time as far as possible into the set of space coordinates but nevertheless insisting on considering time a preferred coordinate by imposing the pseudo-euclidean metric. Such a preference of time is the foundation of the kinematics and dynamics of classical physics.

The revolutionary impression of SRT stems from discarding the Newtonian exclusion logic deeply imprinted in western tradition suggesting space and time to be completely independent entities. Space and time conditioning one another implied the loss of the notion of simultaneity connected with a universal time.

A special relativistic notion seemed to become the prerequisite of any theory in physics, even in General Relativity in a local environment. The only exception is Quantum Mechanics. [97] The advent of General Relativity made the realms of physics based on SRT to become a coast realm of physics being only locally valid as an approximation. The Minkowski metric, the heart piece of SRT, constitutes an absolute object which though acting is not being acted upon. General Relativity by its intrinsic philosophy does not allow for the existence of an absolute object (Norton,1993).

F.3. How elementary particle physics did develop to be tracing the physiognomy of complex flat space

F.3.1. The development of elementary particle physics

Dirac in his search for a relativistic extension of the Schrödinger equation discovers an equation in which the wave functions necessarily are replaced by spinors and the classical vectors p_μ are to be replaced by their associated matrices $\gamma_\mu p^\mu$.

Without any notion of it he enters the world of definitions of Cartan's complex space, in which spinors are the parameters that allow to span this space by isotropic vectors. The γ-matrices take over the role of the reflexion operators that transform the vectors to associated matrices.

He finds spinors the defining equation of which constitutes the massless Dirac-equation which when equipped with mass henceforth is presumed to be the relativistic extension of the Schrödinger equation searched for. As a byproduct the space of Quantum Mechanics, the wave functions of an infinite-dimensional Hilbert space, gets replaced by the familiar 4-dim flat space inhabited by the 4-component Dirac spinors which show a strange transformation profile but live in cohabitation with the four-component classical vectors p_μ.

Extending the space to be complex and equipped with more than four real dimensions he would detect the spinors to have $2^{2\nu}$ components in a space with complex dimension ν the real dimension being $n = 2\nu$ or $n = 2\nu + 1$

But history selected another path. The classical introduction of the e.m. potential A_μ by the replacement $p_\mu \to p_\mu - e/cA_\mu$ led to the notion of generalized gauge invariance replacing the universal gauge by a localized gauge $\Psi \to \Psi e^{i\phi(x)}$ centered around $U(1)$ which with the extension to $SU(2)$ und $SU(3)$ laid the fondament for the success of the Standard Model of elementary particle physics. In this representation the spinors though building the fondament as 2-dim Weyl spinors play

[97]The preferred coordinate of the pseudo-euclidean metric gets identified with *time*. The price to be payed in this special relativistic framing is the requirement that time be imaginary. This has become the obstacle that hinders Quantum Mechanics to adapt to special relativity. To be a measurable entity time would have to be represented by a hermitian operator. As long as time is imaginary this proves to be not possible.

an underpart whereas rotations in form of unitary symmetries are the dominant actors embossing the view of bosons as the generators of unitary symmetries.

The search for a higher symmetry that could provide a common representation for fermions and bosons then led to $SU(5)$ whose representations provided a home for fifteen fundamental particles thereby easily neglecting the never seen right-handed neutrino. The experimental detection of neutrino oscillations giving a mass to the neutrino and therefore asking for the existence of right- and left-handed neutrinos led to replace $SU(5)$ by its cover $SO(10)$. These orthogonal rotations in contrast to unitary transformations provide for spinor representations, which by the now famous 16^+ representation provide home for the 16 fundamental leptons and quarks known today and with all the intrinsic distinctions clicking in place.

By realizing that the interaction Hamiltonians postulated in the Standard Model for the e.m., the weak and the strong interaction essentially mimic the Cartan invariants $\xi^T C \underset{(p])}{X} \xi$ it is rather obvious that this historical development led us right away into the representation of a complex flat space as unfolded by Cartan in 1938.

Obviously the internal consistency of a complex flat space provided the invisible hand which after having got access to 4-component spinors by trial and error (Dirac) enabled to grasp this space concept as a whole.

F.3.2. A long journey: finally discovering the spinor structure

In his 2016 book *Group theory in a nutshell for physicists* Zee presents an overview on the historical development of elementary particle theory.

It began with the historical accident of the Dirac equation in 1928 which for the first time discovered for electrons a representation by spinors. Till then electrons were represented in electrodynamics as the charged point source of an electromagnetic potential. Since 1925 an identification with the mathematical support of the generators of translations combined with Galilei transformations flashed up which increasingly got large credibility in the framework of Quantum Mechanics (see below). The stabilization of the identification with spinors had to wait till the advent of QED in Schweber's Introduction to QED (1962) which changed the quantum mechanical view to become a hybrid of Quantum Mechanics and a quantum field theory. The growing influence of symmetries fired by the success of generalized gauge invariance allowed to widen the view to a theatre of elementary particles dominated by unitary symmetries, $U(1)$, $SU(2)$, $SU(3)$, $SU(5)$.

Zee in detail describes the seesaw of these representations, which obstruct a representation by spinors, till the advent of $SO(10)$, which allowed for the 16^+ spin representation.

F.3.3. Spinor 16^+ seems to be the answer to the quest for unification

The enduring quest for unification has led to probe several higher symmetries. Wilczek (2006) underlines that to allow for a more extensive symmetry the quarks and leptons must furnish the material for building unified structures that remain coherent under the extended symmetry. He notes:

> "One particular unified symmetry passes this test with flying colors. Although the smallest simple group into which $SU(3) \otimes SU(2) \otimes U(1)$ could possibly fit is $SU(5)$ - it fits all the fermions of a single family into two representations $(\mathbf{10} + \mathbf{5})$ and the hypercharges click into place - a larger symmetry group, $SO(10)$ fits these and one additional $SU(2) \otimes SU(2) \otimes U(1)$ singlet particle into a single representation (the spinor $\mathbf{16}$). All 15 quarks and leptons appear on the same footing and the additional particle, which has the quantum numbers of a right-handed neutrino, is quite welcome: it plays a crucial role in the attractive 'seesaw' model for neutrino masses."(Wilczek 2006,242 (written 2003))

It turns out that spin(16) viz. the $1/2 \times 2^\nu$-dimensional semi-spinor representation related to $\nu = 5$ in fact does decompose into all the representations that are required for the Standard Model and its appropriate extension (sect.F.3.3,p.F.3.3): $16^+ \rightarrow 1 \oplus 10 \oplus 5^*$. So the spinor representation contains what was expected and hoped for already by using $SU(5)$.

The spinor representations of $SO(2\nu)$ indeed have the exponential dependence characterizing its dimension to be $dim(SO(2\nu)) = 2^\nu$, as we know it from Cartan. For the space dimension $2\nu = 10$ being even this leads to an irreducible representation in form of semi-spinors with dimension $2^{\nu-1}$ (sect.3.3.2, p.16). Thus for $\nu = 5$ this results in the now famous spinor 16^+, which assembles all particles including the antineutrino in one representation, all of them with the correct quantum numbers.

Wilczek concludes:

> ”Where before we had a piecemeal accomodation of the observed particles, now we have a marvellous correspondence between reality and a unique, ideal mathematical object.”(Wilczek 2006,242)

We conjecture that the ideal mathematical object behind the spinor is the space concept of Cartan which once detected delivers the spinors and their interactions as well.

We conclude: Maxwell’s electrodynamics as well as the develoment of elementary particle theory are a perfect example of how measuring consists in identifying the measuring entities with the variables of a space concept.

F.4. The birth of a new space concept

In general the space structure is determining the objects we are dealing with. Observations and the outcome of experiments might enforce the transition to a new space concept. Observations that resist explanation indicate that the space concept in use is too restrictive. The restriction may concern fundamental ingredients of the space concept requiring for a fundamental change of concept as in General Relativity. Or it may indicate that there are features of the space concept in use hitherto not exploited but rendering itself perceivable. This is what happened with the appearance of the other realms of modern physics, e.g. Quantum Mechanics, each representing an activation of another hidden feature of flat space.

F.4.1. The birth of General Relativity

Two indicators prompted Einstein to switch to another space concept:

- The perihelion shift of Mercury known for centuries and resisting explanation within a Newtonian frame of thought pattern prompted Einstein to skip the severe restrictions that had been imposed on classical physics by using the *rigid* coordinate systems of Euclidean geometry requiring closed orbits in marked contrast to the observed perihelion shift.

- The equivalence of gravitational and inertial mass observed during the acceleration of free fall prompted Einstein to skip the restriction on uniform velocities intrinsic to Galilean and Lorentz transformations when considering the relativity of coordinate systems.

The transition to *Riemannian space* allowed to get independent of the use of coordinate systems by using tensors, tensor equations staying covariant against arbitrary coordinate transformations.

The transition hence was not enforced by detecting and exploiting some new property of a hypothesized Nature. It was induced by getting aware of and skipping a preconception in the own mind of physicists judging as relevant the rigidity of a coordinate system and the preference of uniform velocities. The Riemannian space concept is not a more complicated one favored by Nature but is a simpler one skipping unnecessary restrictions.

From now on a gravito-inertial field which we did not know before the advent of Einsteins General Relativity became identified with the metric $g_{\mu\nu}$ of Riemannian space.

F.4.2. The birth of Quantum Mechanics

Classical mechanics before 1900 was dominated by the deep rooted prejudice that Nature does not proceed via jumps (”Natura non fecit saltus”). A space concept postulating its invariance against translations, rotations and Galilei transformations appeared to be sufficient to guide the explanation of mechanical phenomena by using the contravariant entities energy, momentum and angular momentum for description. The covariant complement of this space concept, indispensable for a process that is exploiting by means of *measuring*, seemed to play no role.

This covariant aspect is embodied in the Lie-algebra of translations spanned by the generators ω and $\vec{k}$. Incorporating Galilei-transformations in this covariant representation a parameter μ enters the covariant stage characterizing an inequivalence of Galilei transformations which we easily recover in its contravariant twin, the inertial mass m_0. The generators combine to form a dispersion relation and a commutator both of which we a posteriori recognize to be the covariant equivalent of the Schrödinger equation and the commutator of Quantum Mechanics (Jauch, 1968). Till here this is a completely classical description of the covariant aspects of a homogeneous space concept incuding Galilei-invariance.

The Schrödinger equation and the commutator of Quantum Mechanics appear on stage as soon as the contravariant measuring entities $E,\ \vec{p}, m_0$ are identified with their covariant counterparts $\omega,\ \vec{k},\ \mu$ by multiplying the latter with $\hbar$:

$$E = \hbar\omega \tag{230}$$

$$\vec{p} = \hbar\vec{k} \tag{231}$$

$$m_0 = \hbar\mu \tag{232}$$

The historical birth of Quantum Mechanics was heralded by the successive discovery of these identifications enforcing the stepwise dissolution of the physicist's prejudice that Nature doesn't proceed by jumps and that particles and waves are separated by an exclusion principle.

It began with the deep rooted inability to model the *black body spectrum* on classical grounds and the startling discovery by Planck that eq.(230) would solve the quandary. The insight that light appeared to occur in the form of quanta appeared on the horizon.

The surprising phenomenon of the *photoelectric effect* five years later led Einstein to propose a solution by eq.(230) supporting the interpretation by light quanta. In 1924 de Broglie made the bold step to associate not only waves with quanta but also particles with waves. Waves and particles from now on were conditioning one another instead of being separated by an exclusion principle. Schrödinger's equation in 1925 then completed the birth of Quantum Mechanics as a self-contained theory based on a space concept determined by the generators of groups of translations combined with Galilei transformations.

The example shows observations being elucidated within the framework of a space concept to lighten corners of the concept previously concealed by prejudice.

F.4.3. The birth of elementary particle physics

When Dirac 1928 in an attempt to break Quantum Mechanics's resistance against its reformulation as a special relativistic theory by trial and error introduced *spinors* into the theory nobody sensed this indicating the transition to a new concept of space taken to be complex and being spanned by isotropic vectors. Spinors are the parameters needed to span the vector space by these vectors of length zero. In his analysis of flat space taken to be complex and defined by reflections the mathematician Cartan in 1938 exposed its structure in details which today we may recognize to reproduce the standard model of elementary particle physics in all its ramifications as soon as we are identifying the leptons and quarks with the spinor components in flat spaces with various higher dimensions.

Elementary particle physics is the paradigm of a realm of physics whose objects together with their interactions come into the world as soon as a new space concept allows to define a condition of the possibility to measure. Leptons and quarks identified with the spinor components get measured as soon as the experiments confirm the condition of the possibility to measure. This condition is given by the Cartan invariant playing the role of a measuring stick and based on the defining equation of spinors which in four dimensions when inverted turns out to be the Dirac equation.

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